

Intercriteria Analysis and Assessment of the Pollution Index for the Bulgarian Section of the Struma River

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Key Words: Struma River; pollution index; intercriteria analysis; index matrices; intuitionistic fuzzy sets.

Abstract. A method called Intercriteria Analysis (ICrA) is applied for analysis and assessment of pollution indices in ten station points to measure the pollution in the Bulgarian section of the Struma River. The following pollution indices have been reviewed: Temperature, pH, Dissolved oxygen, Oxygen saturation, Conductivity, BOD, Permanganate oxidation, Ammonia nitrogen, and Nitrate nitrogen of the Struma River catchment area. The application of the method showed that there is no relationship between the pollution indices (we have a dissonance) in the river catchment area for most of the investigated stations. It is necessary to develop mathematical models for the indices of pollution regarding investigated stations in which there is dissonance. For those stations where there is a partial positive consonance it is not necessary to apply the models.

1. Introduction

The Multi-Criteria Decision Analysis method, called Intercriteria Analysis [3], is based on the apparatus of the index matrix [1] and intuitionistic fuzzy sets [2]. The approach employs the concept of index matrices (IMs), making particular use of some of the operations introduced to them. It also uses the concept of fuzziness of the intuitionistic fuzzy sets thus allowing us to construct the IMs of intuitionistic fuzzy pairs [4], defining the presence or absence of dependency/correlation between any pair of criteria within the set. The ICrA allows the comparison of some criteria or of the objects estimated by them.

Up to now ICrA has found some successful applications into EU member states competitiveness analysis [6,7] and bioprocess modelling [12,14,17,19,20,22].

In [11] ICrA is used for structural and parametric identification of the *Escherichia coli* fed-batch cultivation model and in [21] it presents an improvement of functional state local models of *Escherichia coli* fed-batch cultivation.

In [10] ICrA is attached to establish the basic links between the pollution indicators and to develop mathematical models on this ground. The results show that the criteria (indicators of pollution) are independent and have time functions. Based on that, an adequate mathematical model of pollution of the Mesta River has been developed.

In [13] ICrA is applied to establish relationships between pollution indices based on different criteria. Research has shown that there are three positive consonants and dissonances between the criteria. Using the Modification of the Time Series Method, an adequate mathematical model of the

dynamics of pollution has been developed as a function of time.

The same indices of contamination of the Struma River have been investigated through ICrA in the paper [18].

The aim of this study is the application of ICrA for nine pollution indices for contamination analysis of Struma River in the Bulgarian section in ten stations along the river. This would allow us exclude indices from modelling of the pollution in those stations where we have a positive out ranking (positive consonance).

2. Material and Methods

The transboundary Struma River flows in the western part of Bulgaria and has a catchment area of 107.97 km² and length of 290 km. The catchment follows a mountain pattern and is characterized by a relatively low forestation level. The river water sources are in the high mountain part of the Vitosha and Rila Mountains. The Struma River flows through Bulgaria and Greece to the Aegean.

The transboundary Struma River is located in the western part of Bulgaria and Greece. Its spring is near to the peak Cherni Vrah in the Vitosha Mountain.

The Struma River catchment area is the second largest catchment area in Bulgaria. It is shown in figure 1.



Figure 1. Catchments of the Struma River in Bulgaria

We have investigated physicochemical indices (*common*): C_1 – temperature, C_2 – pH; C_3 – Dissolved Oxygen; C_4 – Oxygen Saturation; C_5 – Electrical conductivity, and *biogenic indices*: C_6 – Biological oxygen demand (BOD₅); C_7 – Permanganate oxidation; C_8 – Ammonia nitrogen, and C_9 – Nitrite nitrogen [8].

The information used is for the period from 2001 to 2005 and was received by the West Aegean Water Basin Directorate, Ministry of Environmental and Water of Bulgaria [16].

These indices were investigated at ten stations in the Struma River catchment area (*table 1*).

3. Intercriteria Analysis Method

The theoretical framework of the ICrA firstly applied by [3] is based on two fundamental concepts: index matrices [1] and intuitionistic fuzzy sets [2].

Intuitionistic fuzzy sets (IFSs) first defined by Atanassov [2] represent an extension of the concept of fuzzy sets, as defined by Zadeh [23].

The difference between the fuzzy sets and intuitionistic fuzzy sets is in the presence of a second function $v_A(x)$ defining the non-membership of the element x to the set A :

$$A = \{ \langle x, \mu_A(x), v_A(x) \rangle \mid x \in E \}$$

where $\mu_A(x), v_A(x): E \rightarrow [0, 1]$ respectively represent the membership and non-membership functions under the condition of $0 \leq \mu_A(x) + v_A(x) \leq 1$.

Table 1. Survey points for water quality analysis of the Struma River

| Point | Point name |
|-------|----------------------------------------------------|
| P50 | The Struma River near Batanovtsi town |
| P80 | The Struma River near Nevestino village |
| P90 | Dzerman River and its infusion in the Struma river |
| P105 | The Struma River before Blagoevgrad town |
| P120 | The Struma River near Krupnik village |
| P122 | Lebnitsa River near Nidukin village |
| P123 | Lebnitsa River near Lebnitsa village |
| P135 | Stumeshnitsa River near Stumeshnitsa village |
| P140 | Stumeshnitsa River after Petrich town |
| P150 | Struma River near the border with Greece |

Comparison between elements of any two IFSs, say A and B , involves pair-wise comparisons between their respective elements' degrees of membership and non-membership to both sets [4].

Here we will start with the index matrix \mathbf{M} and index sets of m rows $\{O_1, \dots, O_m\}$ and n columns $\{C_1, \dots, C_n\}$, where for every $i, j, (1 \leq i \leq m, 1 \leq j \leq n)$, O_i is an evaluated object, C_j is an evaluation criterion, and a_{O_i, C_j} is the evaluation of the i -th object against the j -th criterion which is defined as a real number or another object that in respect to relation R is comparable to the rest elements of the index matrix \mathbf{M} .

$$\mathbf{M} = \begin{array}{c|ccc} & C_1 & \dots & C_n \\ \hline O_1 & a_{O_1, C_1} & \dots & a_{O_1, C_n} \\ \vdots & \vdots & \ddots & \vdots \\ O_m & a_{O_m, C_1} & \dots & a_{O_m, C_n} \end{array}$$

Considering the requirement for comparability reviewed above, it follows that for each i, j, k it holds the relation $R(a_{O_i, C_k}, a_{O_j, C_k})$. The relation R has a dual relation \bar{R} , which is true in the cases when relation R is false, and vice versa. For example, if R is the relation '>', then \bar{R} is the relation '<', and vice versa.

For the needs of our decision making method, pair-wise comparisons between every two different criteria are made along all evaluated objects. During the comparison, it is maintained that one counts for the number of times which the relation R holds, and another counts for the dual relation.

Let $S_{k,l}^\mu$ be the number of cases in which $R(a_{O_i, C_k}, a_{O_j, C_l})$ and $R(a_{O_j, C_l}, a_{O_i, C_k})$ are simultaneously satisfied. Let also $S_{k,l}^\nu$ be the number of cases in which $\bar{R}(a_{O_i, C_k}, a_{O_j, C_l})$ and its dual $\bar{\bar{R}}(a_{O_j, C_l}, a_{O_i, C_k})$ are simultaneously satisfied. As the total number of pair-wise comparisons between the object is $m(m-1)/2$, it might be noticed that the three of them hold the inequalities:

$$0 \leq S_{k,l}^\mu + S_{k,l}^\nu \leq \frac{m(m-1)}{2}$$

For every k, l such that $1 \leq k < l \leq m$, and for $n \geq 2$ the following two numbers are defined:

$$\mu_{C_k, C_l} = 2 \frac{S_{k,l}^\mu}{m(m-1)}, \quad \nu_{C_k, C_l} = 2 \frac{S_{k,l}^\nu}{m(m-1)}$$

Obviously, both $\langle \mu_{C_k, C_l}, \nu_{C_k, C_l} \rangle$ are numbers in the $[0, 1]$ -interval, and their sum is also a number in this interval. What complements their sum near one is the number π_{C_k, C_l} which corresponds to the degree of uncertainty.

The pair, constructed from these two numbers, plays the role of the intuitionistic fuzzy evaluation of the relations that can be established between any two criteria C_k and C_l . In this way the index matrix \mathbf{M} that relates to the evaluated objects with evaluating criteria can be transformed to another index matrix \mathbf{M}^* that gives the following relations among the criteria:

$$\mathbf{M}^* = \begin{array}{c|ccc} & C_1 & \dots & C_n \\ \hline C_1 & \langle \mu_{C_1, C_1}, \nu_{C_1, C_1} \rangle & \dots & \langle \mu_{C_1, C_n}, \nu_{C_1, C_n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_n & \langle \mu_{C_n, C_1}, \nu_{C_n, C_1} \rangle & \dots & \langle \mu_{C_n, C_n}, \nu_{C_n, C_n} \rangle \end{array}$$

In practice, it has been more flexible to work with two index matrices \mathbf{M}^μ and \mathbf{M}^ν , rather than with the index matrix \mathbf{M}^* of intuitionistic fuzzy pairs (IFPs).

The final step of the algorithm is to determine the degree/level of the correlation between the criteria, depending on the user's choice μ and ν . We call these correlations

between the criteria: positive out ranking (*positive consonance*), negative out ranking (*negative consonance*) or dissonance (*there is no relationship between the criteria*).

Let $\alpha, \beta \in [0, 1]$ be the threshold values against which we compare the values of μ_{C_k, C_l} and ν_{C_k, C_l} . We named these criteria C_k and C_l in the following equations:

- (α, β) – *positive consonance*, if $(\mu_{C_k, C_l} > \alpha)$ and $(\nu_{C_k, C_l} < \beta)$;
- (α, β) – *negative consonance*, if $(\mu_{C_k, C_l} < \beta)$ and $(\nu_{C_k, C_l} > \alpha)$;
- (α, β) – *dissonance*, otherwise.

Obviously, the larger α and/or the smaller β , the fewer criteria may be simultaneously connected with the relation of (α, β) – *positive consonance*. It carries the most information when either the *positive* or the *negative consonance* is as large as possible, while the cases of *dissonance* are less informative and therefore for practical reasons, they are skipped.

3.1. Rules for Determining the Degrees of Consonance and Dissonance

Atanassova et al. [5], has discussed an important aspect of the ICrA approach related to the possibilities for defining the intuitionistic fuzzy threshold values that help discriminate between the positive consonance (PC), the negative consonance (NC), and the *dissonance* (D) between the criteria (*figure 2*).

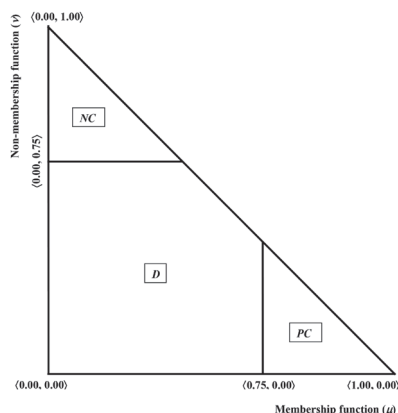


Figure 2. The triangle of *positive consonance*, *negative consonance* and *dissonance*

The triangular zone for the NC from *figure 2* corresponds to the area where the pairs of the criteria which exhibit NC will be located. Formally, this area can be expressed as:

$$NC = \{(\mu, \nu) | \mu \in [0.00, 0.25] \& \nu \in [0.75, 1.00]\}$$

The triangular zone for PC from *figure 2* corresponds to the area where the pairs of criteria which exhibit PC will be located.

Formally, this area can be expressed as:

$$PC = \{(\mu, \nu) | \mu \in [0.75, 1.00] \& \nu \in [0.00, 0.25]\}$$

The pentagonal zone for D from *figure 2* corresponds to the place where the pairs of criteria which are in D will be located. Formally, this area can be expressed as:

$$D = \{(\mu, \nu) | \mu \in [0.00, 0.75] \& \nu \in [0.00, 0.75]\}$$

Since the experimental data of the river pollution is random values for determination of the positive consonance, negative consonance and dissonance the following minimal values are assumed $(\alpha, \beta) = (0.75, 0.25)$.

4. Results and Discussion

The values of membership function (μ) and non-membership function (ν) have been calculated with the help of the software developed by our colleagues [9,15] for the realization of the method.

We have investigated all indices at all measurement points with ICrA. We have searched if there were measurement errors as well as whether the pollution in the different points has had material divergences, as well as if there were dependences between the points.

In this study, we are only interested in membership (μ). Through it, we can determine in which points on the river we have a positive consonance, negative consonance or dissonance for the criteria (stations) examined depending on the selected objects (pollution indices).

The calculated index matrix M_i^μ for the investigated point according to pollution index C_i ($i = 1, \dots, 9$) is shown in *table 2*.

Table 2. Calculated index matrix M_i^μ for the pollution index C_i ($i = 1, \dots, 9$)

| Index matrix M_1^μ for index C_1 (Temperature) | | | | | | | | | |
|------------------------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.834 | | | | | | | | |
| P90 | 0.812 | 0.843 | | | | | | | |
| P105 | 0.811 | 0.770 | 0.764 | | | | | | |
| P120 | 0.812 | 0.768 | 0.756 | 0.933 | | | | | |
| P122 | 0.829 | 0.816 | 0.789 | 0.870 | 0.875 | | | | |
| P123 | 0.721 | 0.698 | 0.711 | 0.774 | 0.774 | 0.843 | | | |
| P135 | 0.814 | 0.783 | 0.779 | 0.849 | 0.833 | 0.919 | 0.847 | | |
| P140 | 0.805 | 0.772 | 0.771 | 0.855 | 0.850 | 0.919 | 0.850 | 0.969 | |
| P150 | 0.810 | 0.769 | 0.771 | 0.863 | 0.856 | 0.909 | 0.843 | 0.944 | 0.951 |

Table 2. (Continue). Calculated index matrix M_i^μ for the pollution index C_i ($i = 1, \dots, 9$)

| Index matrix M_2^μ for index C_2 (pH) | | | | | | | | | |
|---------------------------------------------|-------|-------|--------------|-------|-------|--------|-------|--------------|-------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.556 | | | | | | | | |
| P90 | 0.544 | 0.530 | | | | | | | |
| P105 | 0.563 | 0.491 | 0.544 | | | | | | |
| P120 | 0.470 | 0.472 | 0.482 | 0.625 | | | | | |
| P122 | 0.451 | 0.487 | 0.494 | 0.665 | 0.634 | | | | |
| P123 | 0.591 | 0.506 | 0.545 | 0.628 | 0.493 | 0.646 | | | |
| P135 | 0.599 | 0.533 | 0.548 | 0.642 | 0.605 | 0.580 | 0.591 | | |
| P140 | 0.457 | 0.537 | 0.433 | 0.548 | 0.533 | 0.591, | 0.549 | 0.613 | |
| P150 | 0.547 | 0.549 | 0.541 | 0.590 | 0.596 | 0.596, | 0.630 | 0.687 | 0.595 |

| Index matrix M_3^μ for index C_3 (Dissolved oxygen) | | | | | | | | | |
|-----------------------------------------------------------|--------------|-------|-------|-------|-------|--------------|--------------|--------------|--------------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.592 | | | | | | | | |
| P90 | 0.521 | 0.719 | | | | | | | |
| P105 | 0.708 | 0.697 | 0.595 | | | | | | |
| P120 | 0.708 | 0.600 | 0.530 | 0.698 | | | | | |
| P122 | 0.670 | 0.721 | 0.621 | 0.692 | 0.727 | | | | |
| P123 | 0.648 | 0.717 | 0.632 | 0.727 | 0.719 | 0.859 | | | |
| P135 | 0.673 | 0.698 | 0.614 | 0.741 | 0.746 | 0.863 | 0.873 | | |
| P140 | 0.633 | 0.586 | 0.573 | 0.651 | 0.700 | 0.760 | 0.784 | 0.779 | |
| P150 | 0.673 | 0.700 | 0.613 | 0.725 | 0.721 | 0.867 | 0.856 | 0.851 | 0.805 |

| Index matrix M_4^μ for index C_4 (Oxygen saturation) | | | | | | | | | |
|------------------------------------------------------------|-------|-------|--------------|--------------|-------|--------------|--------------|--------------|-------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.552 | | | | | | | | |
| P90 | 0.562 | 0.732 | | | | | | | |
| P105 | 0.675 | 0.616 | 0.494 | | | | | | |
| P120 | 0.662 | 0.463 | 0.379 | 0.695 | | | | | |
| P122 | 0.617 | 0.513 | 0.406 | 0.756 | 0.689 | | | | |
| P123 | 0.622 | 0.497 | 0.438 | 0.700 | 0.705 | 0.830 | | | |
| P135 | 0.675 | 0.495 | 0.430 | 0.757 | 0.690 | 0.797 | 0.792 | | |
| P140 | 0.584 | 0.467 | 0.419 | 0.606 | 0.637 | 0.714 | 0.738 | 0.710 | |
| P150 | 0.648 | 0.584 | 0.508 | 0.752 | 0.643 | 0.803 | 0.814 | 0.776 | 0.724 |

| Index matrix M_5^μ for index C_5 (Conductivity) | | | | | | | | | |
|-------------------------------------------------------|-------|-------|-------|--------------|--------------|--------------|--------------|--------------|-------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.521 | | | | | | | | |
| P90 | 0.468 | 0.652 | | | | | | | |
| P105 | 0.569 | 0.629 | 0.620 | | | | | | |
| P120 | 0.563 | 0.664 | 0.692 | 0.833 | | | | | |
| P122 | 0.495 | 0.634 | 0.619 | 0.658 | 0.659 | | | | |
| P123 | 0.533 | 0.545 | 0.580 | 0.629 | 0.646 | 0.778 | | | |
| P135 | 0.492 | 0.491 | 0.511 | 0.590 | 0.568 | 0.665 | 0.649 | | |
| P140 | 0.527 | 0.495 | 0.515 | 0.635 | 0.607 | 0.766 | 0.764 | 0.803 | |
| P150 | 0.545 | 0.665 | 0.694 | 0.791 | 0.839 | 0.698 | 0.739 | 0.626 | 0.671 |

Table 2. (Continue). Calculated index matrix M_i^u for the pollution index C_i ($i = 1, \dots, 9$)

| Index matrix M_6^u for index C_6 (BOD) | | | | | | | | | |
|--------------------------------------------|--------------|-------|-------|--------------|-------|-------|-------|-------|-------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.528 | | | | | | | | |
| P90 | 0.508 | 0.476 | | | | | | | |
| P105 | 0.514 | 0.548 | 0.513 | | | | | | |
| P120 | 0.539 | 0.514 | 0.575 | 0.763 | | | | | |
| P122 | 0.434 | 0.417 | 0.497 | 0.612 | 0.583 | | | | |
| P123 | 0.407 | 0.439 | 0.452 | 0.514 | 0.479 | 0.672 | | | |
| P135 | 0.445 | 0.504 | 0.472 | 0.610 | 0.575 | 0.573 | 0.560 | | |
| P140 | 0.413 | 0.429 | 0.550 | 0.573 | 0.603 | 0.602 | 0.538 | 0.615 | |
| P150 | 0.459 | 0.450 | 0.546 | 0.629 | 0.592 | 0.692 | 0.634 | 0.640 | 0.649 |

| Index matrix M_7^u for index C_7 (Permanganate oxidation) | | | | | | | | | |
|---------------------------------------------------------------|-------|-------|-------|--------------|--------------|--------------|--------------|--------------|--------------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.581 | | | | | | | | |
| P90 | 0.483 | 0.468 | | | | | | | |
| P105 | 0.492 | 0.465 | 0.444 | | | | | | |
| P120 | 0.489 | 0.473 | 0.497 | 0.829 | | | | | |
| P122 | 0.429 | 0.437 | 0.357 | 0.741 | 0.727 | | | | |
| P123 | 0.419 | 0.451 | 0.365 | 0.719 | 0.700 | 0.892 | | | |
| P135 | 0.429 | 0.468 | 0.365 | 0.733 | 0.692 | 0.798 | 0.786 | | |
| P140 | 0.433 | 0.422 | 0.414 | 0.794 | 0.757 | 0.757 | 0.759 | 0.800 | |
| P150 | 0.505 | 0.463 | 0.424 | 0.740 | 0.722 | 0.798 | 0.803 | 0.773 | 0.800 |

| Index matrix M_8^u for index C_8 (Ammonia nitrogen) | | | | | | | | | |
|---------------------------------------------------------|--------------|-------|-------|--------------|-------|-------|-------|-------|-------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.518 | | | | | | | | |
| P90 | 0.545 | 0.532 | | | | | | | |
| P105 | 0.498 | 0.500 | 0.449 | | | | | | |
| P120 | 0.592 | 0.571 | 0.574 | 0.641 | | | | | |
| P122 | 0.324 | 0.395 | 0.321 | 0.446 | 0.320 | | | | |
| P123 | 0.297 | 0.398 | 0.372 | 0.459 | 0.324 | 0.640 | | | |
| P135 | 0.485 | 0.435 | 0.459 | 0.425 | 0.426 | 0.500 | 0.482 | | |
| P140 | 0.387 | 0.611 | 0.536 | 0.465 | 0.514 | 0.512 | 0.474 | 0.544 | |
| P150 | 0.477 | 0.599 | 0.471 | 0.577 | 0.494 | 0.488 | 0.521 | 0.628 | 0.605 |

| Index matrix M_9^u for index C_9 (Nitrate nitrogen) | | | | | | | | | |
|---------------------------------------------------------|-------|-------|--------------|-------|-------|--------------|-------|-------|-------|
| Point | P50 | P80 | P90 | P105 | P120 | P122 | P123 | P135 | P140 |
| P80 | 0.508 | | | | | | | | |
| P90 | 0.535 | 0.454 | | | | | | | |
| P105 | 0.430 | 0.483 | 0.494 | | | | | | |
| P120 | 0.594 | 0.519 | 0.589 | 0.533 | | | | | |
| P122 | 0.405 | 0.494 | 0.333 | 0.483 | 0.406 | | | | |
| P123 | 0.465 | 0.471 | 0.375 | 0.425 | 0.462 | 0.763 | | | |
| P135 | 0.695 | 0.516 | 0.446 | 0.497 | 0.525 | 0.497 | 0.522 | | |
| P140 | 0.600 | 0.571 | 0.540 | 0.497 | 0.610 | 0.433 | 0.468 | 0.679 | |
| P150 | 0.483 | 0.457 | 0.460 | 0.563 | 0.646 | 0.457 | 0.478 | 0.508 | 0.533 |

Now let us see *table 2* for membership function (μ) of pollution indices in ten station:

➤ **Index C_1 :** The membership function (index matrix M_1^μ) is changed in the interval $\mu \in [0.698, 0.969]$. *Positive consonance* we have for all points, except P123–P50–P80–P90 points, where we have *dissonance*. Therefore, temperature may be neglected in the modelling.

➤ **Index C_2 :** The membership function (index matrix M_2^μ) is changed in the interval $\mu \in [0.433, 0.687]$. Regarding this index we have *dissonance* at all points. According to C_2 , it is therefore necessary to develop models at all river stations.

➤ **Index C_3 :** The membership function (index matrix M_3^μ) is changed in the interval $\mu \in [0.521, 0.873]$. We have *positive consonance* ($\mu \geq 0.75$) for index C_3 at points: P122–P123–P135–P140–P150. For all points we have *dissonance*. In this case it is necessary to develop models only for stations P50 - P90 - P105 - P120.

➤ **Index C_4 :** The membership function (index matrix M_4^μ) is changed in the interval $\mu \in [0.379, 0.830]$. We have a more complex situation for this index: we have a *positive consonance* at points P105–P122–P135–P150, P122–P135–P150, and P123–P135–P150. For the other points we have *dissonance*. Although we have a positive consonance for some of the stations, it is better to develop models for each station along the river.

➤ **Index C_5 :** The membership function (index matrix M_5^μ) is changed in the interval $\mu \in [0.468, 0.839]$. *Positive consonance* we have for: P105–P122–P135–P150, P122–P123–P135–P150, P123–P135–P150, and P135 – P150. For the other point there is *dissonance*. It is the same for C_4 .

➤ **Index C_6 :** The membership function (index matrix M_6^μ) is changed in the interval $\mu \in [0.407, 0.763]$. For all points there is *dissonance*. The exception is only P105–P120 where we have a *positive consonance*. It is necessary to develop models for all indicators at all stations along the valley.

➤ **Index C_7 :** The membership function (index matrix M_7^μ) is changed in the interval $\mu \in [0.357, 0.892]$. For this index we have a similar situation, like the one with C_4 and C_5 . We have *positive consonance* for: P105–P120–P140, P120–P140, and all points from P122 to P150. For the other points there is *dissonance*. In this case, like index C_3 , it is necessary to develop models only for stations P80-P90-P105-P120.

➤ **Index C_8 :** The membership function (index matrix M_8^μ) is changed in the interval $\mu \in [0.297, 0.641]$. For all points we have *dissonance*, therefore, it is necessary to develop models for all 10 stations along the river.

➤ **Index C_9 :** The membership function (index matrix M_9^μ) is changed in the interval $\mu \in [0.333, 0.763]$. The exception is only P122–P123 where we have a *positive consonance*. Consequently, it is also necessary to develop models for all 10 stations along the river bank.

Conclusion

This article reviews the application of a new method for multicriteria decision making called ICRA. It is applied to evaluate the indices of pollution at ten stations along the

Struma River in the Bulgarian section. ICRA gives the opportunity to show the relation between definite criteria and objects. In this study the selected criteria are ten stations along the river. These criteria are analysed by nine indices (objects) for pollution.

ICRA allows the chosen criteria (stations) to be divided into three categories: stations (or only for some of them) where for a given pollution indicator there is a positive relation, i.e. appurtenance function (μ) is in the interval $\mu \in [0.75, 1.00]$; stations where there is a negative relation $\mu \in [0.00, 0.25]$ for all or for some of them and stations where there is no relation $\mu \in [0.00, 0.75]$. This would allow us to reduce the researched stations when modelling the pollution of a given index.

For the period (2001-2005) reviewed the application of the method shows that there is a positive relation for C_1 at all points except for P123–P50–P80–P90 where there is no relation. This shows us that the temperature can be excluded from the modelling of the riverbed.

For indices C_2 (pH), C_6 (BOD), C_8 (*ammonia nitrogen*) and C_9 (*nitrate nitrogen*) there is no relation at all points along the river. Regarding these indices it is necessary to develop mathematical models along the whole riverbed.

Regarding indices C_4 (*oxygen saturation*) and C_5 (*conductivity*) there is a variety of positive and negative relations (*table 2*). Despite of the positive relations at some points it is also necessary to develop mathematical models along the river using these indices.

The results by indices C_3 (*dissolved oxygen*) and C_7 (*permanganate oxidation*) are interesting as there is no relation for them at stations P50 – P120. It deserves mentioning the points where there is a positive relation (P122 - P150). If we look at *table 1*, we will see that they refer to rivers that flow into the Struma River. Therefore these rivers do not affect the pollution of the river and they can be ignored at the modelling stage.

The next stage of the study is to model these pollution indices along the river where there is a negative relation at all evaluated points. Some of the contemporary methods will be used for the modelling such as neuron network or summarized networks.

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