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Application of the Fractional Algorithms of Control for Discreet Control of a Pneumatic Positioning Device

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Key Words: *Fractional control; discreet fractional repetitive control; repetitive control.*

Abstract. *The work offers fractional repetitive control system of pneumatic positioning device. To prove the effectiveness of the proposed system they are made time, frequency and robust analysis. After which were made experimental studies of observed filtering capabilities on a real laboratory installation of pneumatic positioning device, which is attached discreet fractional re-petitive control.*

Introduction

Essentially, fractional control systems are dynamic systems in full order. They approximate the properties of hypothetical irrational fractal dynamic systems with rational functions in a limited frequency range. For the approximation range coinciding with the operating range of the concerts object control systems, they possess the properties and characteristics of fractal dynamic systems that bring them into the class of the robust control systems.

Significant parts of the industrial objects are characterized by the productivity-induced and/or load-induced changes in the real conditions: repackaging/restructuring of the model; significant inertia and varying delays; the existence of periodic external disturbances whose nominal parameters are known in advance by value but fluctuate operational.

Control systems for objects with similar characteristics should have robust, filtering and invariant properties and variations of delay values, and variations in the parameters of periodic external interference.

In such cases, possible effective control strategy (together with its corresponding principles and systems) with which to realize the control of performance desired objects with characteristics described a strategy repetitive and predictive-repetitive control.

Sampling of the control algorithms is imposed by the fact that they are programmed into controllers, and analog circuits are not used for the realization.

The purpose of the development is:

● Realizing discrete fractional repetitive control on a physical laboratory pneumatic positioning device and **the tasks in its implementation are:**

● Synthesis of fractional repetitive control.

● Time analysis and evaluation of some indicators of performance.

● Robust analysis.

● Discreet realization of fractional control.

● Programming fractional control in PLC.

● Experimental results.

System Control Synthesis

The development is considered as a generalized automation object – a pneumatic positioning device, which is shown in *figure 1*. The generalized automation object is comprised of compressor *K*, proportional distributor valve *V1*, pneumatic actuator *A1*, potentiometer *P*, controller (PLC).

For generalized automation object, the transfer functions of the nominal (1) and perturbed en uppermost limit (2) models are known in advance.

They are known in the literature [11,15] various structures of the repetitive control systems. The work is selected modified structure of repetitive filter. It is suitable for this application study in the control of a proportional feedback pneumatic laboratory stand because it provides an extension of the cut-off frequency band.

The synthesis of repetitive controller in the control system takes place in two stages:

● Designing a base regulator *R(p)*.

● Designing a robust repetitive filter *ML*, which are independent of each other and also independent of the methods of synthesis used.

In the literature [1-4], the systems of non-integer order are known. The structure of the fractional control system is shown in *figure 3*. Used are the following indications: $ML \circ R_{I_{NE}}$ – fractional repetitive regulator in the system; regulator of non-integer order $R_{I_{NE}}$ (3), rationally approximating the behavior of an operator to integrate I^{α} of the generalized fractional calculus from order *0.9* analytical synthesized by the method of recursive polynomial approximation [1,2,15] where α is non-integer order of the operator, and Γ is the gamma function.

In both the proposed two-step synthesis procedure, the solution is expressed in analytical design of the system *figure 2* for fractional control with regulator $R_{I_{NE}}$ (3) and fractional repetitive control system *figure 3* with a regulator $ML \circ R_{I_{NE}}$ (6).

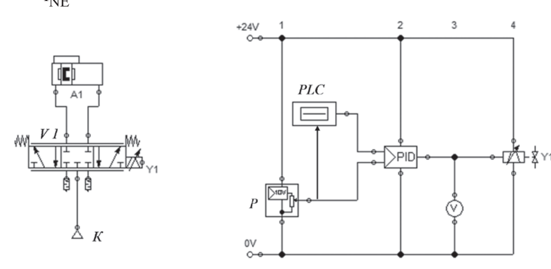


Figure 1

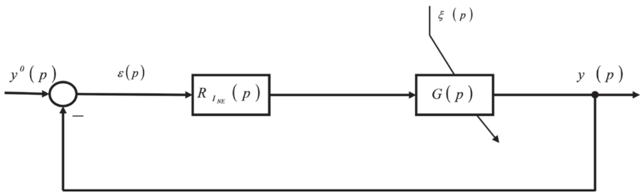


Figure 2

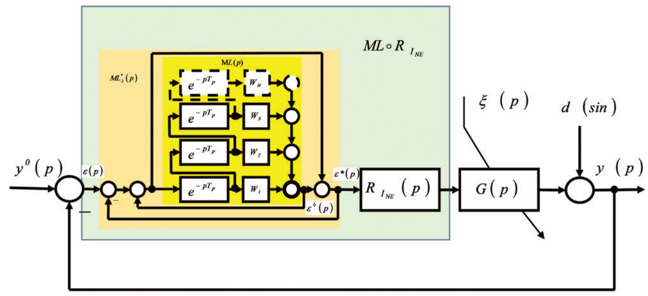


Figure 3

$$(1)\quad G^*(p)=\frac{y(p)}{u(p)}=\frac{5}{0.0289\,p^2+0.119\,p+1}$$

$$(2)\quad G^{\blacksquare}(p)=\frac{y(p)}{u(p)}=\frac{132.2}{p^2+2.6\,p+26.44}$$

$$(3.1)\quad {}_aI_t^{\alpha} f(t)=\frac{1}{\Gamma(-\alpha)}\int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1}}\,d\tau\ ,\ (\alpha, a\in\Re, \alpha=0.9)$$

$$(3.2)\quad R_{I_{NE}}(p)\equiv I_{app}^{0.9}(p)=0.01747\frac{(17.1\,p+1)}{(24.75\,p+1)}\cdot\frac{(13.48\,p+1)}{(19.91\,p+1)}\cdot\frac{(10.23\,p+1)}{(0.00203\,p+1)}$$

$$(4)\quad t_{reg.(y^o=1(t))}\in\left[t_{0.95},\ t_{1.05}\right],\quad (0{,}95\,h(\infty)<h(t)<1{,}05\,h(\infty))$$

(5)

$$\mathcal{M}_L(p)=\left(2-\sum_{k=1}^{m}W_k(p)e^{-pkT_p}\right)^{-1}\hat{\equiv}\left(2-e^{-pT_p}\right)^{-1},\ \left(k=1;T_p\equiv T_{pf}=2\pi\left(\omega_{pf}\right)^{-1}=2\pi\left(1{,}2\right)^{-1}\right)$$

(6)

$$M_L(p)R^*_{I_{NE}}(p)=0.01747\,\frac{(17.1p+1)}{(24.75p+1)}\cdot\frac{(13.48p+1)}{(19.91p+1)}\cdot\frac{(10.23p+1)}{(0.00203\,p+1)}\cdot\frac{1}{\left(2-e^{-pT_p}\right)}$$

Time Analysis and Evaluation of Some Indicators Performance

The synthesized control systems *figure 2* and *figure 3* are modeled and simulated in MATLAB environment. The simulation results of the models – the time and frequency characteristics of the closed Φ_i and open W_i loop systems for control of object G are visualized in nominal G≙G* (1) parametric mode without the presence of a permanently occurring periodic external signal disturbance d, as follows:

• The transfer function of *figure 4*, which shows that in the reaction of the control system without the filter and the reaction of the system with fractal regulator and repetitive filter it still has an aperiodic character (4).

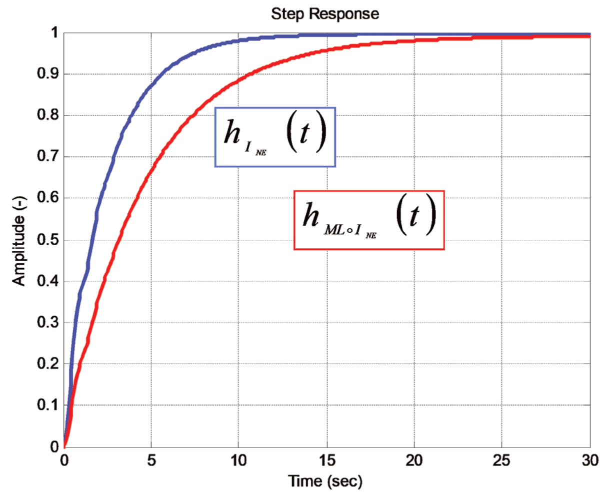
• Frequency characteristics of the open loop systems *figure 5*, from which it is clear that the fractional repetitive $W_{ML\circ I_{NE}}$ control system has a larger gain margin and phase margin (7) than the fractal system $W_{I_{NE}}$.

The analysis of the performance of the systems (*figure 2*, *figure 3*) confirms that both systems:

● Satisfy the generalized performance criteria (*figure 4*, *figure 5*).

● They (*figure 5*) are stable in nominal parameter mode G≙G* (1) (1).

$$(7)\quad \begin{array}{c} GM=20\log_{10}\left|W^*(j\omega_\pi)\right|,[dB];\ PM=-\left(arg\left(W^*(j\omega_0)\right)+180^{\circ}\right),[deg],\\ \left(\omega_\pi : arg\,W^*(j\omega_\pi)\equiv\pi;\ \ \omega_0 : \left|W^*(j\omega_0)\right|\equiv 1\right)\end{array}$$



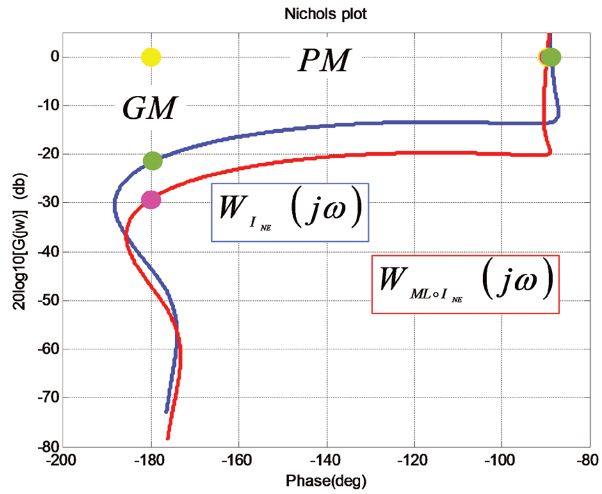


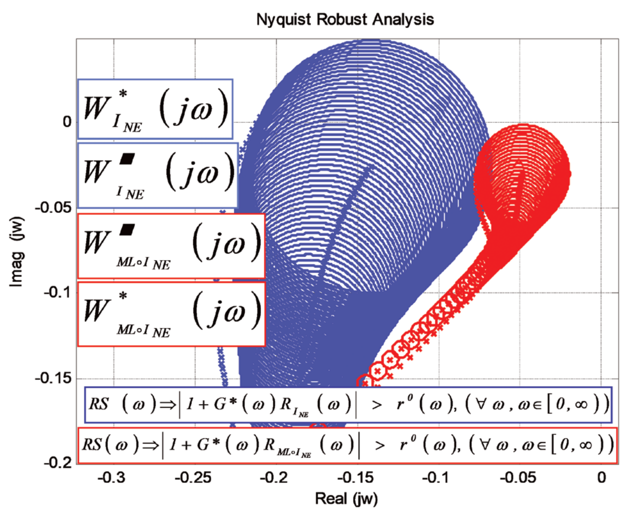
Figure 4

Figure 5

Robust Analysis

Visualized results of frequency 2D Nyquistrobust analysis of the performance for the characteristics of the open loop systems *RS* (10) and *RP* (11) in the characteristics of the open loop, systems also can be determined:

- The margin of robust stability $k_{MSOL}(\omega)$ under (12), *figure 8*.
- The margin of robust performance $k_{MPOL}(\omega)$ under (13), *figure 9*. On the characteristics of closed loop systems, *RS* (8) and *RP* (9), (*figure 7*), systems using sensitivity functions (10) and complementary sensitivity (11). It is obvious that the design systems have proven robust stability RS_i and proven robust performance RP_i in the context of the parametric fluctuation of $G^{\bullet}$ (2) to G^* (1), set forth in the synthesis of systems. The results of the robust analysis on the characteristics of the open loop systems (*figure 6*) and the closed ones (*figure 7*). Analytically prove and uniquely visualized that in the condition of repartition/restructuring ξ, $\xi^{\bullet}$ the nominal model of the object (1) the system without a repetitive filter *ML* (*figure 2*) and the repetitive fractal system (*figure 3*) have robust stability *RS* and robust performance *RP*, as their characteristics meet the requirements formulated by (8-11). The margins of robust stability and robust performance that meet the requirements described in (12 and 13) have also been visualized.



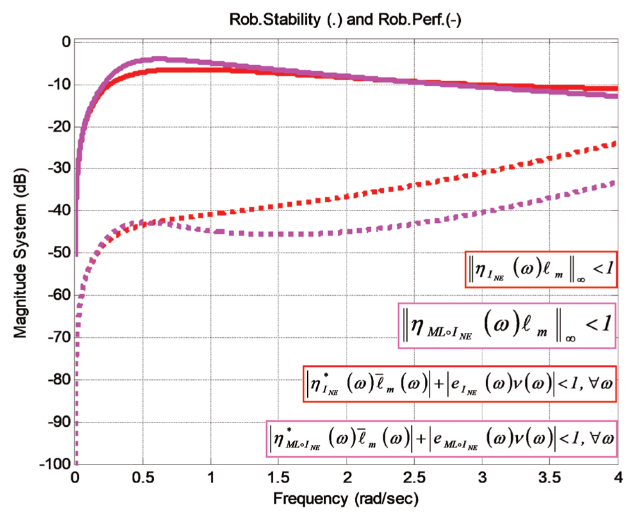


Figure 6

Figure 7

$$(8)\ RS(\omega)\Rightarrow\|\eta(\omega)\bar{\ell}_m(\omega)\|_\infty<1,\forall\omega,(\omega\in[0;\infty)),$$
$$RS(\omega)\Rightarrow|1+R(\omega)G^*(\omega)|>r^0(\omega),\forall\omega$$

$$(9)\ RP(\omega)\Rightarrow|\eta^*(\omega)\bar{\ell}_m(\omega)|+|e^*(\omega)v(\omega)|<1,\forall\omega,(\omega\in[0;\infty)),$$
$$RS(\omega)\Rightarrow|1+G^*(\omega)R(\omega)| > |G^*(\omega)R(\omega)|\ \bar{\ell}_m(\omega),(\forall\omega,\omega\in[0,\infty))$$
$$(10)\ e(\omega)=(1+R^*(\omega)G^*(\omega))^{-1}\equiv\Phi_{y^*\varepsilon}(\omega),(e(\omega)=1-\eta(\omega))$$

$$(11)\quad \eta(\omega)=R^*(\omega)\,G^*(\omega)\left(1+R^*(\omega)G^*(\omega)\right)^{-1} \equiv \Phi_{y^* y}(\omega),\ \left(\eta(\omega)=1-e(\omega)\right)$$

$$(12)\quad k_{MSOL}(\omega)=r^0(\omega)\left|1+R(j\omega)\,G^*(j\omega)\right|^{-1} \le 1,\ \left(\forall\,\omega,\ \omega\in[0,\infty)\right)$$

(13)

$$k_{MSOL}(\omega)=r^0(\omega)\left|1+R(j\omega)\,G^*(j\omega)\right|^{-1} \le 1,\ \left(\forall\,\omega,\ \omega\in[0,\infty)\right)$$

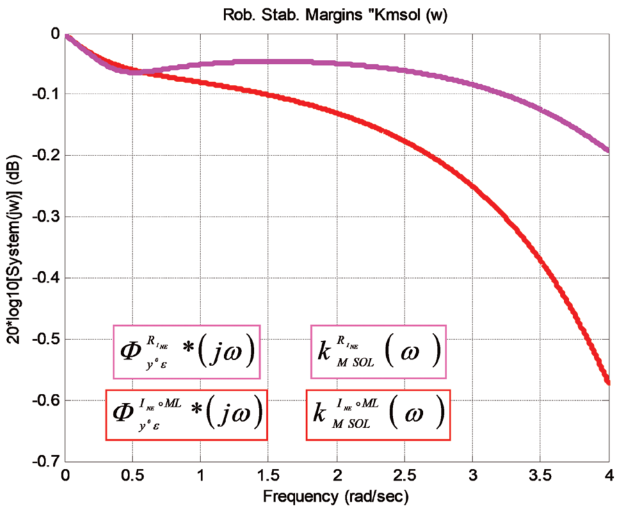


Figure 8

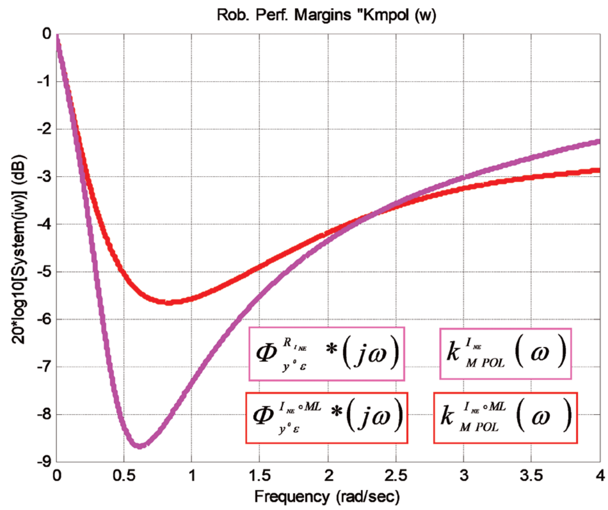


Figure 9

Discrete Realization of Algorithm for Fractal Control

In [3,4,5,6,7,8,9] various methods are known to sampling the rational approximations of the operators of the generalized fractional calculus. For the programming of the synthesized fractional repetitive regulator in the PLC, sampling using the Tustin method with sampling rate is used $T_0=0{,}001s$. For the purposes of sampling, the Tustin transformation uses the Muir-recursion (14.1). The operator's $(1\text{-}z^{-1})^{\pm\alpha}$ approximation to the Padé series is described in (14.2). For comparison in (14.3), Taylor's extended series is shown. Sampling of the fractal repetitive algorithm consists of two stages:

- Sampling of repetitive filter.
- Sampling of non-integer regulator.

Repetitive filter (5) *figure 11* is composed of aperiodic units $W_k(p)$ and units with a delay e^{-pkT_p}. Both are discrete as shown in (15.3) and (15.1), after which their descriptions are brought into differential equations (15.4) and (15.2) respectively with a discrete time k counter which are programmed in the PLC controller. The fractal regulator (3.2) is scaled to (16.1), *figure 12* and converted to differential equation (16.2), which in turn is programmed in the PLC controller. The variables $e1$, $e2$, $e3$, used in *figure 12* and the PLC controller program code shown are equivalent to the variables $e1 \triangleq \varepsilon^*$, $e2 \triangleq \varepsilon_2^*$, $e3 \triangleq \varepsilon_3^*$ used in (14.2).

The implementation of the control algorithm **[12]** was accomplished using a functional repetitive filter unit. This enables the function block, in which the repetitive filter is programmed, to be called after the fractal regulator block has been programmed (*figure 10*). The program code of the repetitive filter is as follows:

```
#e1 := #e - #e2;
#x := #e1 + #z;
#y := 1.995004 * #y1 - 0.995013 * #y2 + #x - 2.004979 * #x1 + 1.004988 * #x2;
#z := 0.367871 * #z1 + 0.632106 * #y1;
#y2 := #y1;
#y1 := #y;
#x2 := #x1;
#x1 := #x;
#z1 := #z;
#e2 := #z + #e1;
#e3 := #e2;
```

For fractional repetitive control $ML \circ R_{I_{NE}}$ program code is as follows:

```
"ML_DB"(e := #sp - #pv,
        e3 => #e);
#e1 := 0.017437 * #e;
#e2 := 0.999998 * #y1_1 + #e1 - 0.9866 * #e1_1;
#y1_1 := #e2;
#e1_1 := #e1;
#e3 := 0.9803 * #y2_1 + #e2 - 0.998 * #e2_1;
#y2_1 := #e3;
#e2_1:=#e2;
#mv := 0.97569 * #y3_1 + #e3 - 0.983 * #e3_1;
IF #mv > 1 THEN
        #mv := 1;
END_IF;
```

$$(14.1)\quad D^{\alpha}\left(z^{-1}\right)\equiv\left(\varpi\left(z^{-1}\right)\right)^{\alpha}=\left(\frac{2}{T_0}\right)^{\alpha}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^{\alpha}$$

(14.2)

$$D^{\alpha}\left(z^{-1},\alpha\right)=\left(\frac{2}{T_0}\right)^{\alpha}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^{\alpha}\approx\left(\frac{2}{T_0}\right)^{-\alpha} PSE\left\{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^{\pm\alpha}\right\}_{Taylor}$$

(14.3)

$$\hat{D}^{\alpha}\left(z^{-1},\alpha\right)=\left(\frac{2}{T_0}\right)^{\alpha}\left(\frac{1-\alpha z^{-1}}{1+\alpha z^{-1}}\right)_{Pade}$$

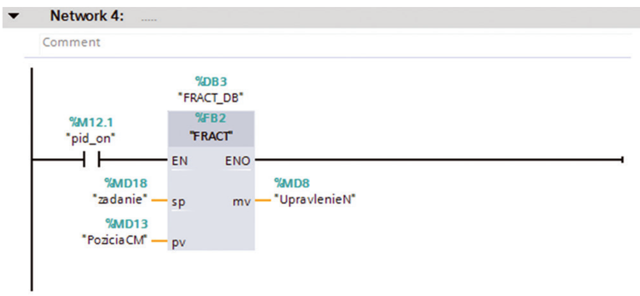


Figure 10

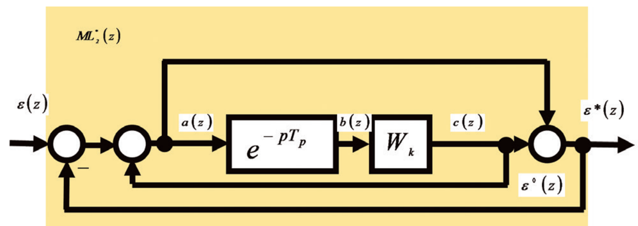


Figure 11

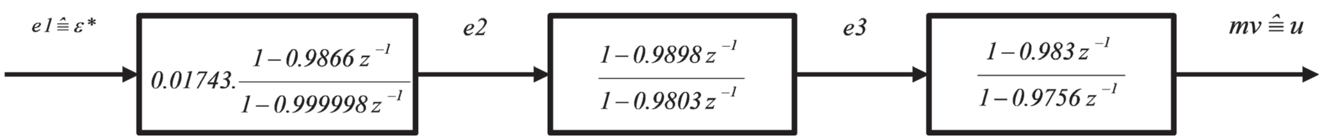


Figure 12

$$(15.1)\quad e^{-zT_p}=\frac{z^2-2.004979z+1.004988}{z^2-1.995004z+0.995013}=\frac{a(z)}{b(z)}$$

$$(15.2)\quad b(k)=1.995004b(k-1)-0.995013b(k-2)+a(k)-2.004979a(k-1)+1.004988a(k-2)$$

$$(15.3)\quad W_k(z)=\frac{0.632106}{z-0.367871}=\frac{c(z)}{b(z)}$$

$$(15.4)\quad c(k)=0.367871\ c(k-1)+0.632106\ b(k-1)$$

$$(16.1)\quad R_{I_{NE}}=0.01743.\frac{\left(1-0.9866z^{-1}\right)}{\left(1-0.999998z^{-1}\right)}.\frac{\left(1-0.9898z^{-1}\right)}{\left(1-0.9803z^{-1}\right)}.\frac{\left(1-0.983z^{-1}\right)}{\left(1-0.9756z^{-1}\right)}=\frac{u(z)}{\varepsilon^*(z)}$$

$$(16.2)\left\{\begin{array}{l}\varepsilon_2^*(k)\triangleq e2(k)=0.999998\,\varepsilon_2^*(k-1)+0.017437\left(\varepsilon^*(k)-0.9866\,\varepsilon^*(k-1)\right)\\ \varepsilon_3^*(k)\triangleq e3(k)=0.9803\,\varepsilon_3^*(k)(k-1)+\varepsilon_2^*(k)-0.9898\,\varepsilon_2^*(k)(k-1)\\ u(k)\triangleq mv(k)=0.9756\,mv(k-1)+\varepsilon_2^*(k)-0.983\,\varepsilon_3^*(k-1)\end{array}\right\}$$

Experimental Results

The results of the actual experiments are as follows:

- *Figure 13.1.* – $h^*_{I_{NE}}$ for three different set point values.
- *Figure 13.2.* – $h^*_{ML\circ I_{NE}}$ for three different set point values.
- *Figure 13.3.* – $h^*_{I_{NE}}$ and $h^*_{ML\circ I_{NE}}$
- *Figure 14.1.* – $h_{I_{NE}}$ for three different set point values.
- *Figure 14.2.* – $h^{\bullet}_{ML\circ I_{NE}}$ for three different set point values;
- *Figure 14.3.* – $h^{\bullet}_{I_{NE}}$ and $h^{\bullet}_{ML\circ I_{NE}}$.

The following indications are used: μ – the position of the pneumatic positioning device; $h^*_{I_{NE}}$ – a step response of fractal system in nominal parametric mode; $h^*_{ML\circ I_{NE}}$ – a step response of a fractal repetitive system in the current parametric mode; $h^{\bullet}_{I_{NE}}$ – a step response of fractal system in perturbed en uppermost limit parametric mode; $h^{\bullet}_{ML\circ I_{NE}}$ – a step response of a fractional repetitive system in a perturbed en uppermost limit parametric mode.

The results of the actual experiments under laboratory conditions confirm the improved properties of the repetitive to the fractional control systems to counteract periodic disturbances caused by friction. Improved performance is expressed in increasing the period of repetitive disturbances and reducing the amplitude of periodic disturbances as shown in *figure 13.3* and *figure 14.3*. From the same figures, it can be seen that the repetitive filter in combination with a fractal regulator leads to an improved system accuracy in an established mode.

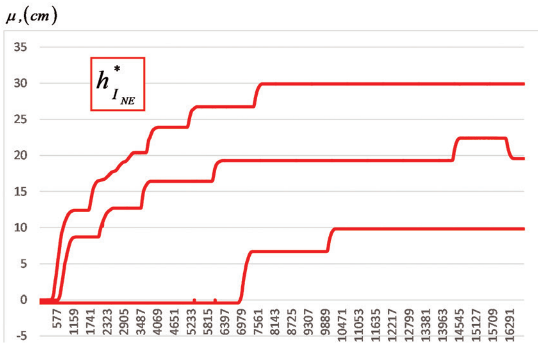


Figure 13.1

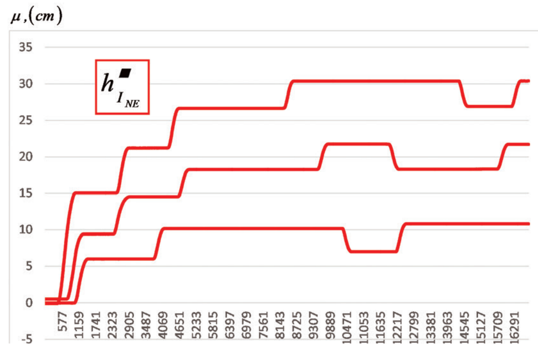


Figure 14.1

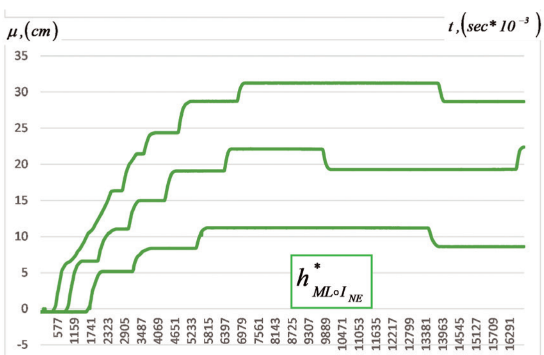


Figure 13.2

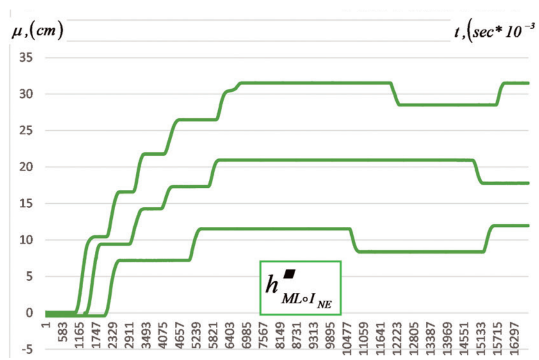


Figure 14.2

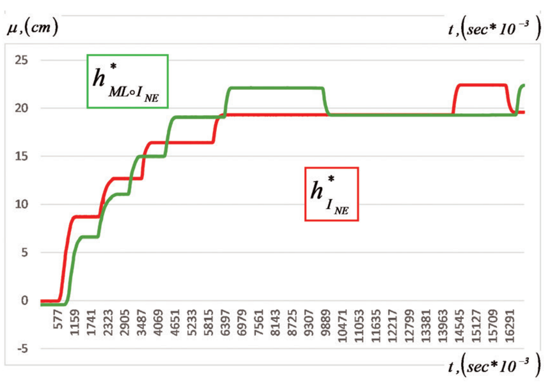


Figure 13.3

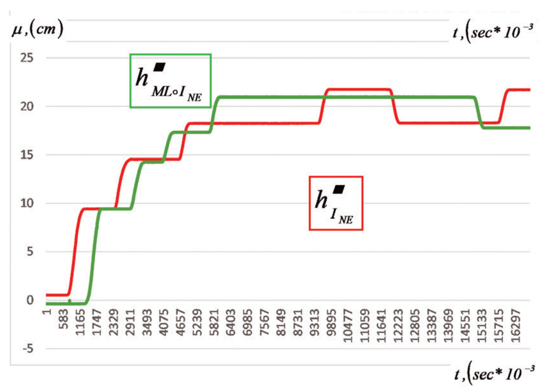


Figure 14.3

Conclusion

The study was directed to the development of discrete frac-tal repetitive control of physical laboratory pneumatic positioning device.

The new and original in the development is that:

• A comparative time analysis was performed.

• Some performance indicators have been evaluated.

• Perform a robust analysis in open and closed systems.

• Margins of robust stability and robust performance have been evaluated.

• It is discretized and PLC programming algorithm for management of non-integer order.

• It is discretized and programmed the PLC repetitive filter.

• Software has been developed in the PLC controller.

• Experimental lab tests have been carried out confirming the advantages of the repetitive to the fractional control sys-tems to counteract periodic disturbances caused by friction.

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