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Embedded LQG Control of Tank Liquid Level

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Key words: *LQG controller; Embedded control system; Liquid level control.*

Abstract. *In this paper the developed low cost embedded system for liquid level control of tank is presented. The plant is a physical laboratory model of a water tank. The liquid level is controlled in wide range by designed LQG controller. The LQG controller is designed on the basis of obtained by identification linear black box model. A software in MATLAB®/Simulink® environment for generation of code which is embedded in low cost control kit ARDUINO Mega 2560 is developed. Simulation and experimental results are given that confirm the workability of the embedded control system.*

Introduction

The control of liquid level of tank is a problem that commonly occurs in many industrial applications. They are mainly related to the food processing, chemical industrial processes, agriculture and nutrition. Due to that the system for automatic liquid level control is one of the popular experimental setup in industrial control and embedded control laboratories [1]. Usually the control of liquid level is performed by PID controller or its modifications. However in presence of significant noise produced by level sensor, power supply and/or actuators the conventional PID controller cannot provide control system performance in whole working range. In these cases advanced control techniques such as LQG and H_∞ control can be used. This motivates authors to design a low cost embedded system for liquid level control based on the LQG controller.

In this paper, development and experimental evaluation of low-cost embedded system for LQG control of liquid level of tank physical model is presented. The plant is a physical laboratory model of water tank produced by Lucas Nulle Company [2]. The liquid level is controlled in wide range by designed LQG controllers. The control algorithm is implemented in low cost control kit ARDUINO Mega 2560 [3]. Software in MATLAB/Simulink environment is developed for generation of control code. Some simple hardware devices are developed too. These devices provide appropriate voltage level of analogue signals which are exchanging between physical model of tank and control kit. Controllers are designed on the basis of the stochastic linear discrete-time black box model derived from experimental data by identification. Results from simulation of the closed-loop system as well as experimental results obtained from real-time implementation of designed controllers are given. They confirm embedded control system performance in whole working range.

Hardware Configuration of Embedded Control System

The scheme of embedded system for liquid level control is shown in *figure 1*. The system is comprised of water tank, voltage divider, DIP reed relay and ARDUINO Mega 2560 kit. The aim of control is to set the water level to desired one regardless amount of water which flows out through the outlet valve. The desired level is set by reference signal r and it is achieved via manipulating inlet water flow by regulating the rotational speed of water pump. The water level is measured by filling level sensor.

To manipulate the water pump rotational speed the voltage PWM signal with high level of 12 V and low level of 0 V is needed. The PWM signal generated by digital pin of *Arduino Mega 2560* is 8-bit and has frequency of 490 Hz, high level of 5 V and low level of 0 V. Thus it should

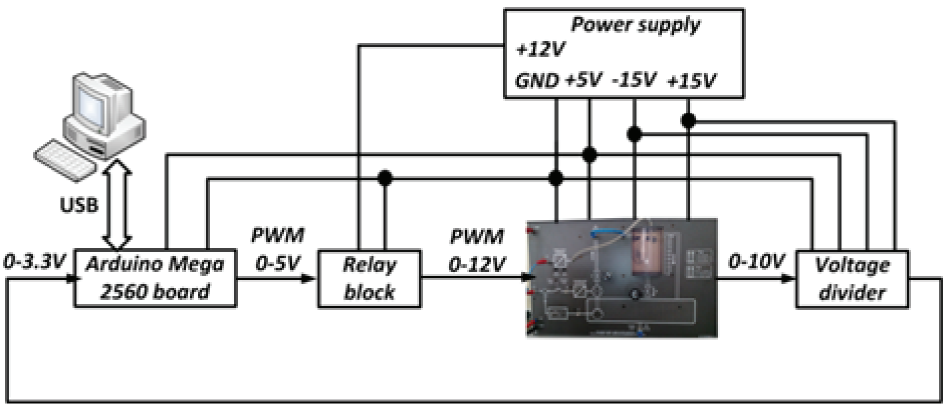

Figure 1. Embedded system for control of liquid level

be amplified to high level of 12 V. This is done by designed by the authors *Relay block.* The control signal of embedded system is integer number which is written to the PWM output. It can take values in range 0-255.

The output signal of water level sensor takes values in range 0-10 V, but the range of ADC converter of ARDUINO Mega 2560 is 0-5 V. Thus the sensor voltage signal should be divided. This is done by the designed by authors voltage divider, which linearly transform the sensor voltage from range of 0-10 V to the range of 0-3.3 V. The 10 bit ADC generates integer numbers in range 0-1023, but the value of voltage divider's output signal which corresponds to the water level of 100% is approximately 3.3 V. This means that the maximum integer number obtained from ADC is 676. Thus the control variable can take integer values in range 0-676.

Plant Identification

To determine the mathematical model of tank one may apply physical modeling or identification. Physical modeling requires profound of knowledge about physics of the plant and a lot of a priori information such as pump characteristics and hydraulic resistance of outlet valve. Due to the lack of a priori information, in this study we prefer to use a numerical model obtained by identification procedure. Another reason to use an identification model is that such model in addition to the tank dynamics takes into account the dynamics of water level sensor and water pump which facilitates the plant description. Moreover by identification we may obtain model of the noise, which can be used to design an appropriate optimal filter such as Kalman filter. The identification procedure starts with measuring of plant static characteristics (*figure 2*). It shows the range in which we can control the water level by manipulating water pump voltage via the 8-bit PWM signal.

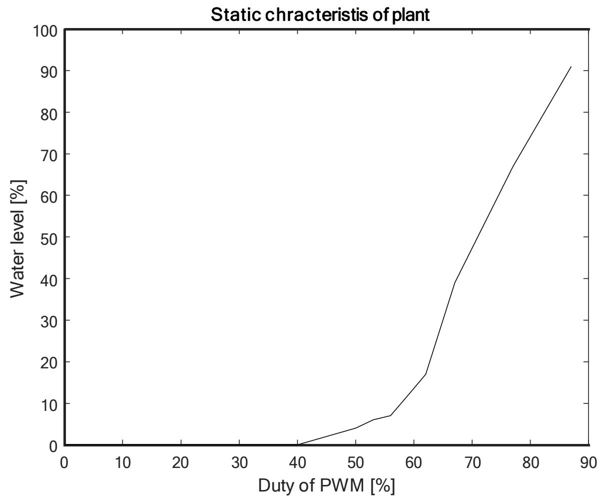

Figure 2. Plant static characteristic

As can be seen from *figure 2* the plant has linear characteristics for values of input signal in range 58-100%. The water pump has wide dead zone of 58%. To use linear plant model and to control water level by linear controller we should avoid the actuator dead zone. This may be done by adding the constant value of 58% to input signal. Static characteristic shows that we can control water level in whole working range with duty coefficients between 59-100%.

To obtain black box model the open loop identification experiments is designed. The sample time Ts=0.5 s is chosen which is sufficiently small. The constant input signal of 75% for first 630 s of the identification experiment is applied. This value is approximately equal to the middle of the actuator linear range. As a result the water level achieves steady state value of approximately 62%. After first 630 s the random binary signal (RBS) is added to constant input signal which provides persistent excitation of identification signal. RBS is obtained from filtered through relay white Gaussian noise. The amplitude of RBS is chosen to be ±15 %, so the input signal takes two values of 60% and 90%. In this manner the whole linear range of input signal is used. The measured input output data are shown in *figure 3*. It is seen that the plant output is corrupted by significant noise. This noise comprises power supply noise, measurement noise and noise generated from A/D conversion of output signal. Before use input output data set for identification the part of data which corresponds to the constant value of input signal is cut and the remaining part is centered. Then two data sets are formed. The first is used for model estimation and the second is used for model validation.

The estimated state space model is

$$(1) \quad \begin{aligned} x(k+1) &= Ax(k) + Bu(k) + K_n e(k) \\ y(k) &= Cx(k) + Du(k) + e(k) \end{aligned},$$

where A, B, C, D, K_n are matrices with appropriate dimensions and $e(k)$ is residual error. Assuming that the model order is between 1 and 5, the five models are estimated. The order of the best model of this set is determined by Hankel singular values which are shown in *figure 4*. As it is seen, the best model order is 1. Then the 1th order state-space model is estimated by prediction error method. The obtained model has matrices

$$A = 0.9942, B = 1.887\times10^{-4}, C = 114, D = 0, K = 2.89\times10^{-4}.$$

To validate estimated model the several validation tests are performed. The results from whitening and independence tests are shown in *figure 5*. The frequency response of estimated high order finite impulse response (FIR) model between control signal and residuals along with 99%confidence region is depicted in *figure 6*. The comparison between measured step response and model step response is presented in *figure 7.*

As can be seen from *figures 5* and *6* obtained model passes the tests, which means that the model captures sufficiently well tank dynamics, the noise model is adequate and there is not significant dynamics between input signal and residuals in the whole interested frequency range. The fit between measured and model step responses is excellent.

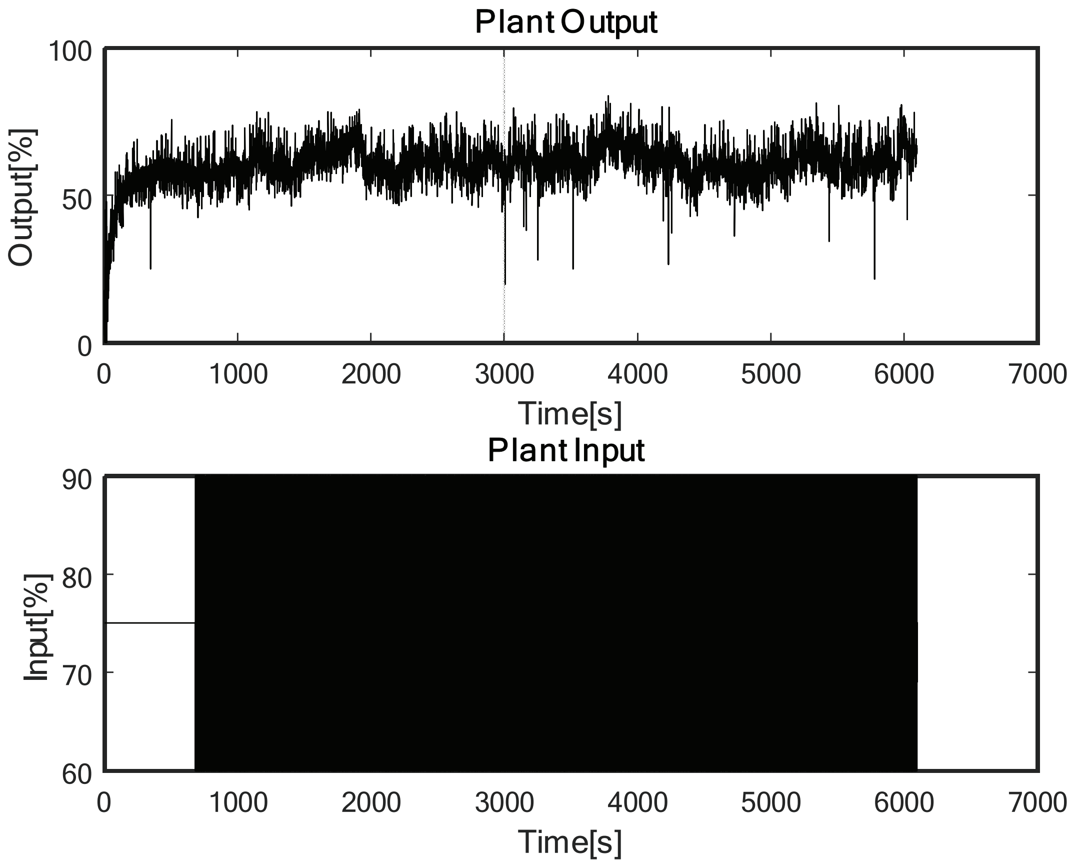

Figure 3. Measured input output data

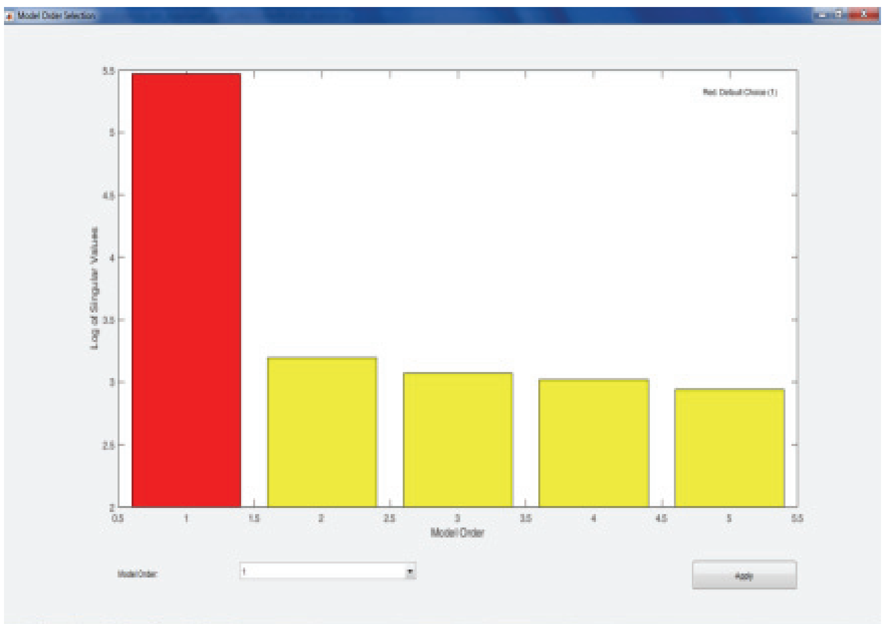

Figure 4. Hankel singular value

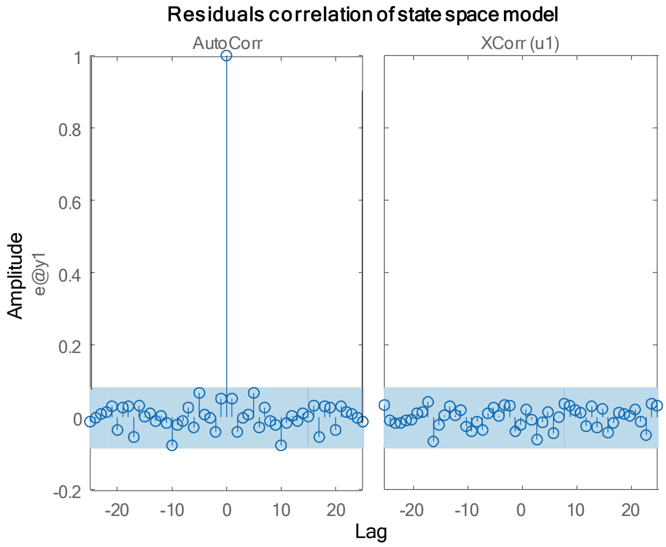

Figure 5. Residual test of state space models

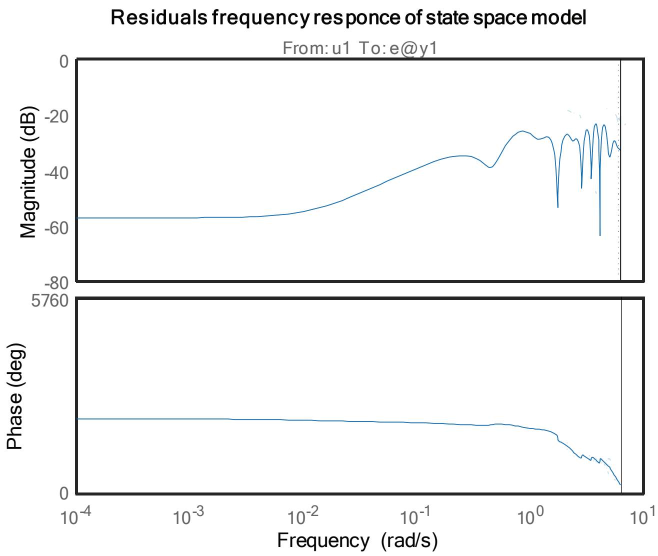

Figure 6. Residual to input signal frequency response

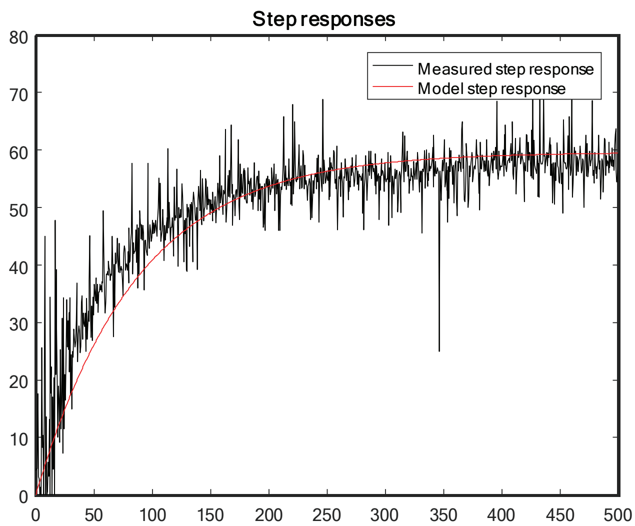

Figure 7. Model step response and measured step response

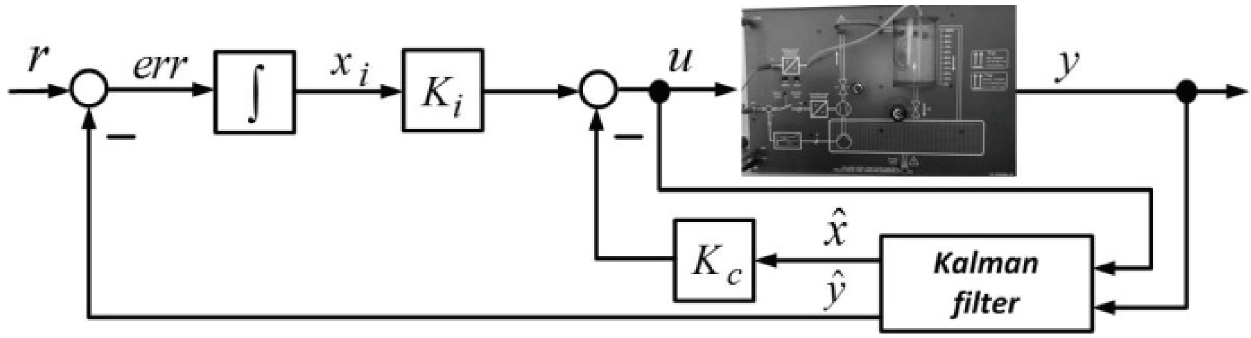

Figure 8. Control system with LQG controller

As a result from identification the 1-th order state space model is obtained. It describes sufficiently well both the plant and the noise dynamics.

LQR/LQG Controller Design

The structure block-diagram of water level control system with LQG controller is shown in *figure 8*.

To include integral action in LQG controller the approximation of discrete time integral of system error is taken in the form

(2) $x_i(k+1) = x_i(k) + T_s err(k) = x_i(k) + T_s(r(k) - y(k))$,

where $x_i(k)$ is the integral of system error $err(k)$, $T_s = 0.5$ *s* is the sample time and $r(k)$ is the reference. Combining equations (1) and (2) one obtains the augmented system

(3) $$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k) + \bar{G}r(k), \quad y(k) = \bar{C}\bar{x}(k),$$

where

$$\bar{x}(k) = \begin{vmatrix} x(k) \\ x_i(k) \end{vmatrix}, \bar{A} = \begin{vmatrix} A & 0 \\ -T_s C & 1 \end{vmatrix}, \bar{B} = \begin{vmatrix} B \\ 0 \end{vmatrix}, \bar{C} = \begin{vmatrix} C & 0 \end{vmatrix}, \bar{G} = \begin{vmatrix} 0 \\ T_s \end{vmatrix}.$$

The optimal LQR control law which minimizes

(4) $$J = \sum_{k=0}^{\infty} [\bar{x}^T(k) Q \bar{x}(k) + u^T(k) R u(k)],$$

takes the form

(5) $u(k) = -\bar{K}\bar{x}(k), \bar{K} = [K_c \quad -K_i]$,

where K_c and K_i are the controller coefficients, Q and R are the positive definite weighting matrices. The controller matrix is obtained from expression

(6) $\bar{K} = (R + \bar{B}^T P \bar{B})^{-1} \bar{B}^T P \bar{A}$,

where matrix P is the positive definite solution of the equation

(7) $\bar{A}^T P \bar{A} - P - \bar{A}^T P \bar{B}(R + \bar{B}^T P \bar{B})^{-1} \bar{B}^T P \bar{A} + Q = 0$.

The design of LQR controller is done for

(8) $$Q = \begin{vmatrix} 1.5 & 0 \\ 0 & CC^T \end{vmatrix}, R = 500.$$

Note that there is not necessity to design a state observer because the model (1) is of first order and the state can be obtained as

(9) $x(k) = C^{-1} y(k)$.

Thus the water level control can be accomplished with simple LQR controller. However the presence of significant noise in the measured output determines a need to reduce influence of noise to water level and especially to actuator. This is done by Kalman filter

$$\hat{x}(k) = (A - K_f CA)\hat{x}(k-1) + (B - K_f CB)u(k-1) + K_f y(k),$$
$$\hat{y}(k) = C\hat{x}(k)$$

where $\hat{x}(k)$ is the state estimate and $\hat{y}(k)$ is the estimate of water level and K_f is the Kalman filter gain. The LQG controller forms control signal according to

(10) $$u(k) = -K_c \hat{x}(k) + K_i \hat{x}_i(k), \quad \hat{x}_i(k+1) = \hat{x}_i(k) + T_s(r(k) - \hat{y}(k)).$$

The magnitude plot of the closed-loop system is shown in *figure 9* and the sensitivity of control signal to model noise is depicted in *figure 10*.

As should have been expected the closed loop system with both controllers has the same characteristics. This is due to that the frequency responses are calculated for zero initial conditions. It is seen that the closed-loop bandwidth is approximately 0.085 rad/s, which is sufficiently larger than plant bandwidth. The closed loop system will track without steady state error reference signal with frequency up to 0.085 rad/s. The control signal of closed loop system with LQR controller is very sensitive to noise in high frequencies. Thus for this system the noise will be amplified by controller. As a result undesirable oscillations with significant amplitude will be occurred in control signal. In real application this may cause damage of actuator. Whereas the control signal for closed loop system with LQG controller is not sensitive to model noise in high frequencies due to the filtration properties of Kalman filter.

Experimental Results

The specialized software in MATLAB/Simulink environment is developed to implement the water level control code for the designed LQG controller. With the aid of Simulink Coder, Embedded Coder, Arduino IDE 1.6.12 and Simulink Support Package for Arduino Hardware v.16.1.2 a

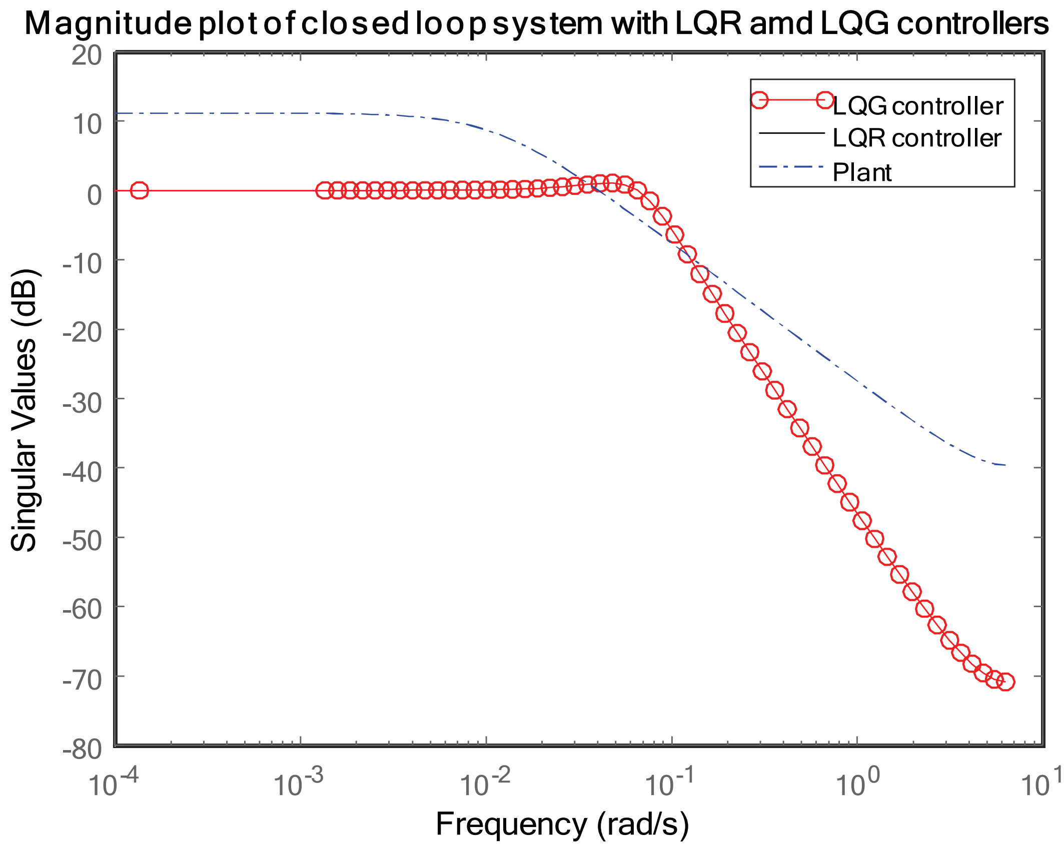

Figure 9. Magnitude plot of closed loop system

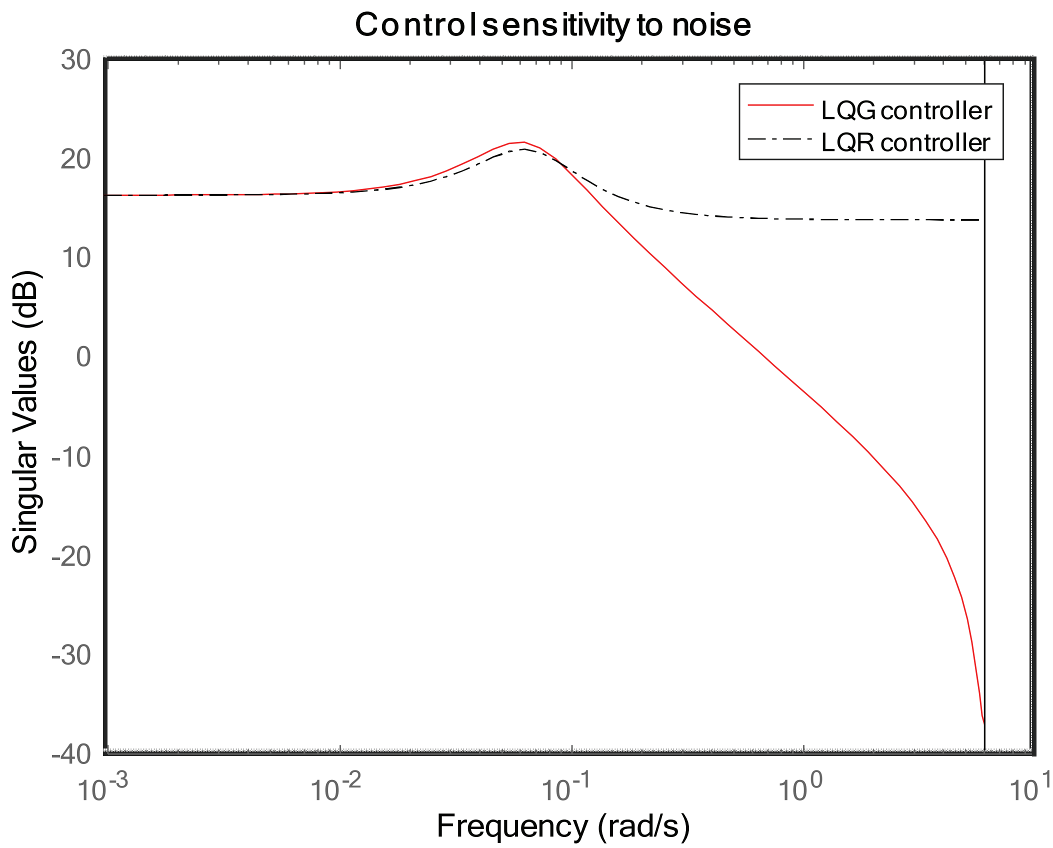

Figure 10. Control signal sensitivity to model noise

code is generated from this software which is embedded in the microcontroller *ATmega2560*. The main advantages of code generation technology are the highly shorten time for coding of control algorithm, reducing the overall time for testing and verification of the developed algorithm, and the relatively easy implementation of complex control algorithms [4]. The software to generate code for water level LQG control is shown in *figure 11*.

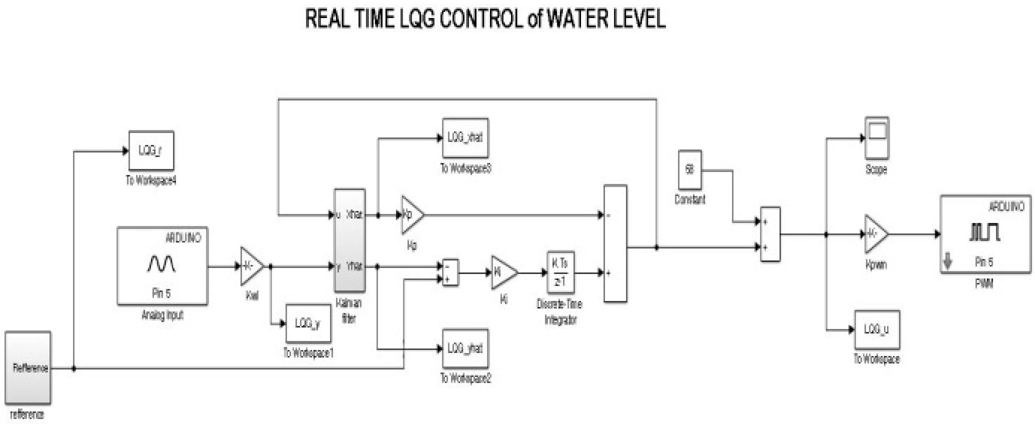

Figure 11. Simulink model for code generation

During the experiment the reference signal is varying in wide working range according to the expression

$$r = \begin{cases} 40\%, 0 \le t < 500 \\ 60\%, 500 \le t < 1000 \\ 80\%, 1000 \le t < 1500 \\ 45\%, 1500 \le t < 2000 \\ 60\%, 2000 \le t < 2500 \end{cases} \quad (11)$$

The estimated by Kalman filter water level from experiment and the water level from simulation are shown in *figure 12*. The control signal from simulation and the control signal from experiment are depicted in *figure 13*. In *figure 14* the measured output and estimated output from experiment are presented.

It is seen that there is a very significant noise in measured output signal. This noise comprises of measurement noise, noise from power supply and noise from ADC conversion. Nevertheless the Kalman filter smooths out very well system output (see *figure 14*), which means that the obtained by identification model is adequate. This statement is confirmed by excellent closeness between output from simulation and estimated output from experiment (see *figure 12*). The control system with LQG controller has very good performance for whole working range. The settling

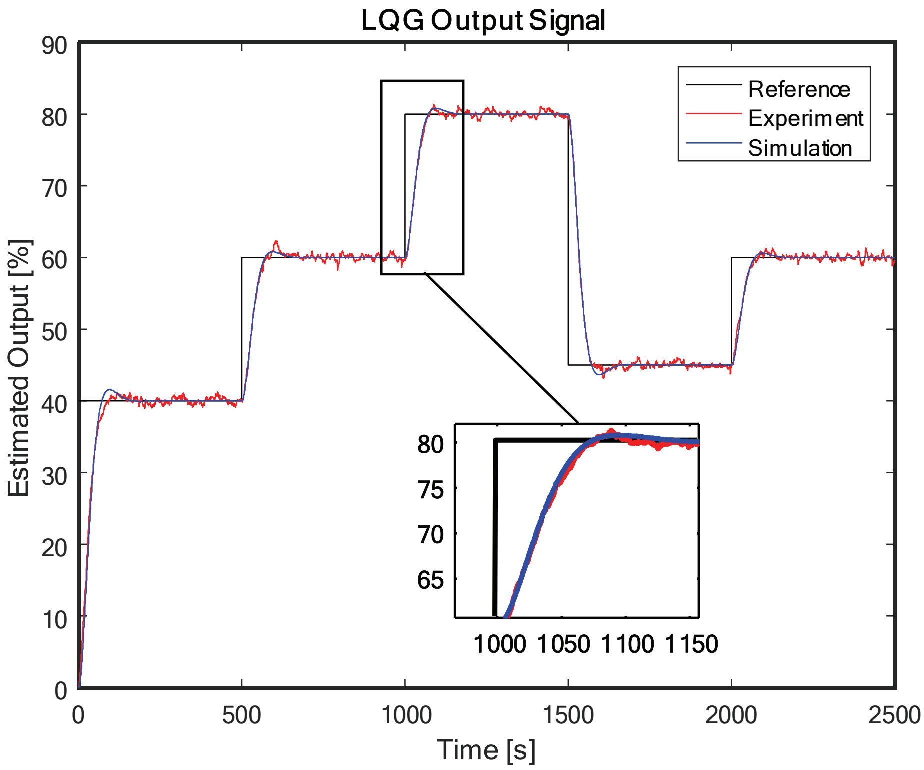

Figure 12. Output signals from experiment and simulation

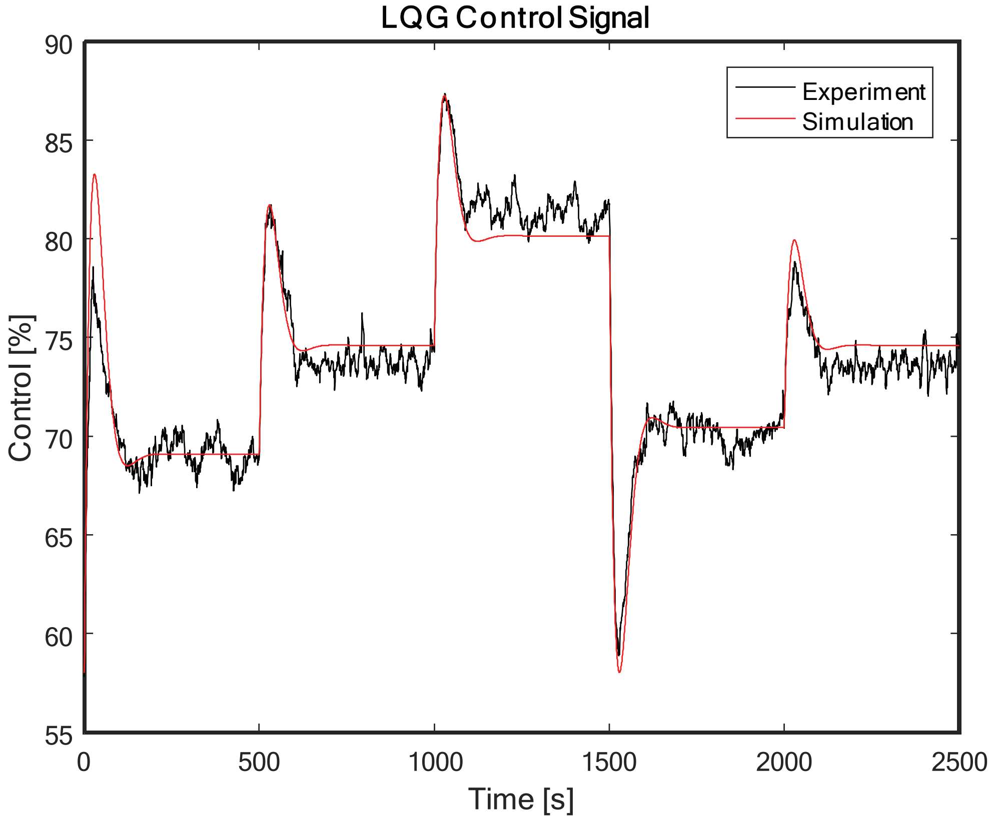

Figure 13. Control signals from experiment and simulation

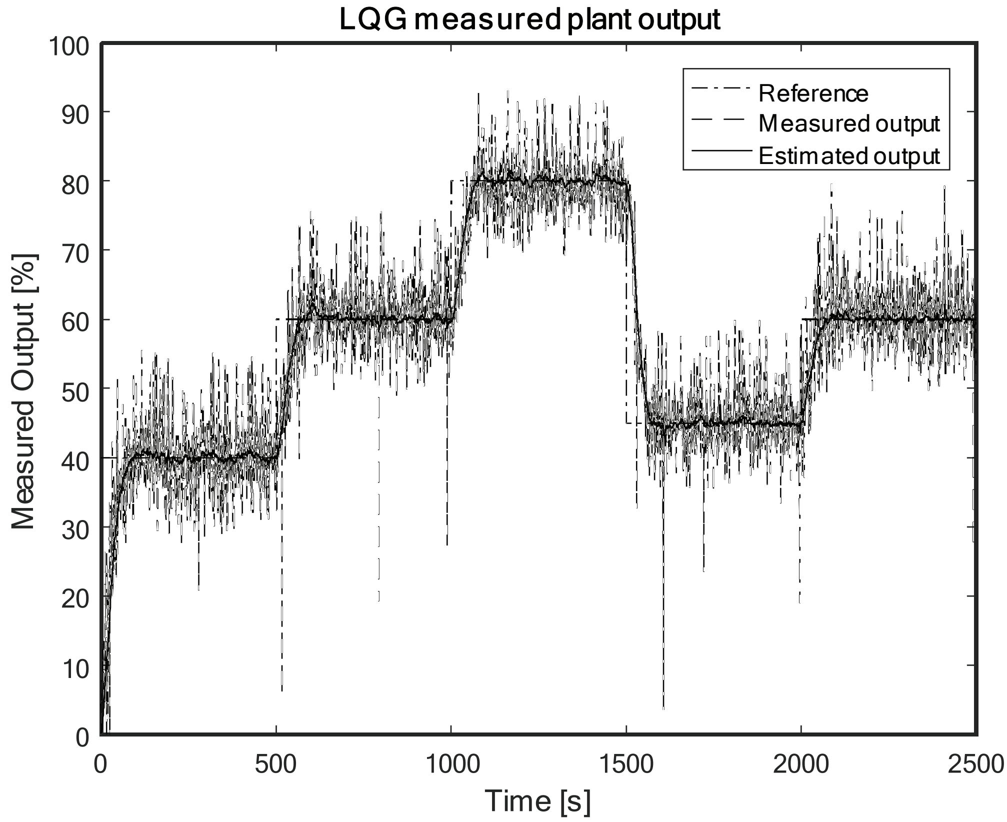

Figure 14. Measured output and estimated output

times of all transient responses depicted in *figure 12* are approximately 70 s, which is 4 times shorter than duration of plant step response. The overshoot is negligible. This is very important for tank, because overshoot means that extra quantity of water will flow through tank. As a result from filtration of output signal and plant state the control signal has sufficiently small oscillations. Thus the actuator works properly and safely. The control signal does not achieve maximum value of 100%.

Conclusion

In this paper the developed low cost embedded system for liquid level control is presented. The designed LQG controller provides control system performance in presence of significant noise and the output signal tracks very well the reference signal in whole working range. The experimental results confirm a workability of embedded control system. The designed LQG controller is successfully embedded in low-cost microcontroller. This confirms that the sophisticated control laws can be easy implemented in low-cost microcontrollers.

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