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Traffic Management of Urban Network by Bi-level Optimization

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Key Words: *Traffic control; transportation behavior; urban network of crossroad traffic lights; hierarchical approach; bi-level optimization.*

Abstract. *A city network of crossroad sections is under consideration in order to reduce the traffic jams, the traffic queue lengths in front of the junctions, and to increase the outgoing traffic flows. The implementation of these goals is achieved by application of hierarchical approach. A bi-level optimization is applied for finding the optimal control parameters as solutions of appropriate optimization problems, hierarchically interconnected. The numerical simulations' results show improvement of the traffic's characteristics.*

1. Introduction

The traffic management is wide discussed problem for many years because of its importance in our everyday life. The actuality of this topic is caused of its complexity and attempts of the world's scientific society to be applied the modern theoretical and simulation achievements [1, 2]. An analysis of the traffic control strategies is presented in [3, 4]. The main control parameters, used for the traffic management in urban areas are the traffic light cycle duration, the green light duration of the traffic light cycle, and the traffic lights offset in a network of crossroads. Usually, the presented papers concern optimization of only one of these control parameters. Optimization problem using only one control parameter (the green light duration) is considered in [5] for the design of traffic lights plans and for minimization of the queue lengths in front of the traffic lights based on the store-and-forward model in [6]. Optimal signal settings only for the traffic lights are considered in [7-9].

The above researches aim solutions of one criterion optimization problems. In this paper we are solving two-criterion optimization problem based on the hierarchical system's theory. Because of the complexity of the multilevel manner of optimization, practical application has the bi-level approach where two-level hierarchical system is considered. The bi-level optimization extends the optimization environment (goal, parameters and constraints) by the embedded philosophy of its functionality through interconnections between the both optimization problems.

2. Bi-level Optimization

The bi-level optimization targets optimization of two interconnected optimization problems. The solution of the lower-level problem is sent to the upper level problem where this optimal solution is regarded as a parameter for the upper level optimization problem. The analogical is the interconnection between the upper and lower level optimization problem. This hierarchical approach is chosen in our research because it allows smaller optimization problems to be solved easier on each level and because of their interconnections more complex optimization problem is solved. In practice, instead of two independent sub-problems, one global problem with two goals is solved. This complex problem has more parameters (the optimization variables of the lower and upper level optimization problems), larger number of constraints (the resources of the both optimization problems) and satisfaction of two goal functions. The interconnection between the both optimization problems leads to optimization of more goals, satisfying more constraints and including more parameters in comparison with the classical optimization of two optimization problems twice. This tendency of the application of the bi-level optimization is analyzed in [10, 11]. Nevertheless the difficulties of the usage of the bi-level approach, it is applied in different domains nowadays because of its triple advantages, mentioned above. The bi-level optimization is applied in [12] for logistic purposes minimizing the customers' costs and satisfying their demands. Another logistic problem is solved by bi-level optimization in [13] where *m* sources have to be distributed to n destinations. Different priorities are assigned to the destinations by hierarchical manner. In [14] the bi-level optimization is applied for locating of logistics.

In [15] the bi-level approach is applied to the public transport. The time interval among the different buses is optimized on the upper level having in mind their capacities. The lower level determines the user's preferences for routes. Similar problem is considered in [16] where the upper level minimizes the travel costs and lower level – bus transportation scheme.

This short overview shows that the bi-level optimization has bigger instrumentation to consider more requirements, which is the main reason to be applied for decision making problems, management of logistics problems and transportation. These advantages of the bi-level approach are due to the inclusion of more parameters, constraints and goals of these interconnected optimization problems. For these reasons the bi-level approach is applied in this research for improving the traffic characteristics of urban network of crossroads.

3. Determination of the optimization problems

The architecture of the city network is presented in *figure 1*. We consider a network with five crossroad sections and the goal is to decrease the queue lengths in horizontal direction from West to East and vise versa where is the main traffic flow in Sofia. For the network the traffic lights cycles are c_j, $j = 1, \ldots, 5$. It includes green, amber and red light. The amber light is 0.1 of the traffic light cycle. The traffic light cycle is c_1 for the first crossroad section. The green light duration for the first junction is u_1 in vertical direction. For the horizontal direction the green light duration is $(0.9c_1 - u_1)$. The queue lengths are x_i, $i = 1, \ldots, 18$, *figure 1*. We suppose that for the first crossroad section the saturations are S_1 in horizontal and S_2 in vertical direction. For the next four junctions the saturations are respectively $S_3 - S_4$, $S_5 - S_6$, $S_7 - S_8$, and $S_9 - S_{10}$. The distance and density between the first and second traffic lights are respectively L_1 and ρ_1. For the next three parts they are respectively L_2 and ρ_2, L_3 and ρ_3, L_4 and ρ_4. The outgoing flows are q_k, $k = 1, \ldots, 8$, *figure 1*.

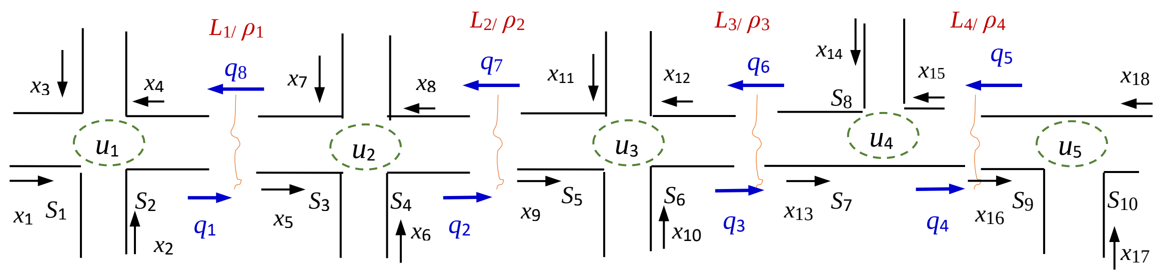

Figure 1. Urban traffic network

3.1. Determination of the lower-level optimization problem

The goal of the lower-level problem is to minimize the queue lengths in front of the traffic lights. We choose a quadratic optimization goal function with arguments queue lengths and the green light durations of the all five junctions of the network. The constraints of this problem are based on the store – and – forward model, applied for the all 18 queue lengths. The lower-level problem is

(1) $$\min_{\substack{i=1,\ldots,18\\ j=1,\ldots,5}} (x_i^2 + u_j^2)$$

subject to

(2) $$x_1 - S_1u_1 + 0.9S_1c_1 \le x_{1_0} + x_{1_{in}}$$

$$x_2 + S_2u_1 \le x_{2_0} + x_{2_{in}}$$

$$x_3 + S_2u_1 \le x_{3_0} + x_{3_{in}}$$

$$x_4 - S_1u_1 + S_3u_2 + 0.9S_1c_1 - 0.9S_3c_2 \le x_{4_0}$$

$$x_5 + S_1u_1 - S_3u_2 - 0.9S_1c_1 + 0.9S_3c_2 \le x_{5_0}$$

$$x_6 + S_4u_2 \le x_{6_0} + x_{6_{in}}$$

$$x_7 + S_4u_2 \le x_{7_0} + x_{7_{in}}$$

$$x_8 - S_3u_2 + S_5u_3 + 0.9S_3c_2 - 0.9S_5c_3 \le x_{8_0}$$

$$x_9 + S_3u_2 - S_5u_3 - 0.9S_3c_2 + 0.9S_5c_3 \le x_{9_0}$$

$$x_{10} + S_6u_3 \le x_{10_0} + x_{10_{in}}$$

$$x_{11} + S_6u_3 \le x_{11_0} + x_{11_{in}}$$

$$x_{12} - S_5u_3 + S_7u_4 + 0.9S_5c_3 - 0.9S_7c_4 \le x_{12_0}$$

$$x_{13} + S_5u_3 - S_7u_4 - 0.9S_5c_3 + 0.9S_7c_4 \le x_{13_0}$$

$$x_{14} + S_8u_4 \le x_{14_0} + x_{14_{in}}$$

$$x_{15} - S_7u_4 + S_9u_5 + 0.9S_7c_4 - 0.9S_9c_5 \le x_{15_0}$$

$$x_{16} + S_7u_4 - S_9u_5 - 0.9S_7c_4 + 0.9S_9c_5 \le x_{16_0}$$

$$x_{17} + S_{10}u_5 \le x_{17_0} + x_{17_{in}}$$

$$x_{18} - S_{91}u_5 + 0.9S_9c_5 \le x_{18_0} + x_{18_{in}}.$$

Problem (1) can be presented in vector form like

(3) $\min\limits_{x,u}(x'Q_1x + u'Q_2u)$

subject to

(4) $A_1x + A_2u + A_3y \le B$,

where Q_1, Q_2, A_1 are unit matrices:

(5) $Q_1 = eye(18{,}18)$, $Q_2 = eye(5{,}5)$,

$A_1 = eye(18{,}18)$,

(6) $$A_2 = \left\| \begin{matrix} -S_1 & 0 & 0 & 0 & 0 \\ S_2 & 0 & 0 & 0 & 0 \\ S_2 & 0 & 0 & 0 & 0 \\ -S_1 & S_3 & 0 & 0 & 0 \\ S_1 & -S_3 & 0 & 0 & 0 \\ 0 & S_4 & 0 & 0 & 0 \\ 0 & S_4 & 0 & 0 & 0 \\ 0 & -S_3 & S_5 & 0 & 0 \\ 0 & S_3 & -S_5 & 0 & 0 \\ 0 & 0 & S_6 & 0 & 0 \\ 0 & 0 & S_6 & 0 & 0 \\ 0 & 0 & -S_5 & S_7 & 0 \\ 0 & 0 & S_5 & -S_7 & 0 \\ 0 & 0 & 0 & S_8 & 0 \\ 0 & 0 & 0 & -S_7 & S_9 \\ 0 & 0 & 0 & S_7 & -S_9 \\ 0 & 0 & 0 & 0 & S_{10} \\ 0 & 0 & 0 & 0 & -S_9 \end{matrix} \right\|$$

(7) $$A_3 = \left\| \begin{matrix} 0.9S_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.9S_1 & -0.9S_3 & 0 & 0 & 0 \\ -0.9S_1 & 0.9S_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9S_3 & -0.9S_5 & 0 & 0 \\ 0 & -0.9S_3 & 0.9S_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9S_5 & -0.9S_7 & 0 \\ 0 & 0 & -0.9S_5 & 0.9S_7 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9S_7 & -0.9S_9 \\ 0 & 0 & 0 & -0.9S_7 & 0.9S_9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9S_9 \end{matrix} \right\|$$

(8) $$B = \left\| \begin{matrix} x_{1_0} + x_{1_{in}} \\ x_{2_0} + x_{2_{in}} \\ x_{3_0} + x_{3_{in}} \\ x_{4_0} \\ x_{5_0} \\ x_{6_0} + x_{6_{in}} \\ x_{7_0} + x_{7_{in}} \\ x_{8_0} \\ x_{9_0} \\ x_{10_0} + x_{10_{in}} \\ x_{11_0} + x_{11_{in}} \\ x_{12_0} \\ x_{13_0} \\ x_{14_0} + x_{14_{in}} \\ x_{15_0} \\ x_{16_0} \\ x_{17_0} + x_{17_{in}} \\ x_{18_0} + x_{18_{in}} \end{matrix} \right\|$$

3.2. Determination of the upper-level optimization problem

The upper level optimization problem aims maximization of the outgoing from the crossroad section traffic flow. This model is based on the one of the main transportation model – the continuity of the traffic flow, which formalization is below. The traffic flow is proportional to the traffic flow speed and traffic density and for the first traffic flow between the first and second junction it is:

(10) $q_1 = v\rho_1$.

The traffic flow speed is

(11) $v = v_{free}\left(1 - \frac{\rho_1}{\rho_{1max}}\right)$.

Substituting (11) in (10), it is obtained:

(12) $q_1 = v_{free}\left(1 - \frac{\rho_1}{\rho_{1max}}\right)\rho_1.$

The density ρ_1 is formalized as relation between the number of cars on the distance between the first and the second traffic lights

(13) $\rho_1 = x_5/L_1.$

After substitution of (13) in (12) it is obtained

(14) $q_1 = \frac{v_{free}}{L_1}\left(x_5 - \frac{x_5^2}{\rho_{1max}L_1}\right).$

As the constant v_{free}/L_1 does not influence the optimization, we can ignore it. For simplification we denote

(15) $\beta_1 = \frac{1}{\rho_{1max L_1}}$.

Then (14) can be written in the form

(16) $q_1 = (x_5 - \beta_1 x_5^2)$.

Analogically, we can present the rest seven outgoing flows like

(17) $q_2 = (x_9 - \beta_2 x_9^2)$

$q_3 = (x_{13} - \beta_3 x_{13}^2)$

$q_4 = (x_{16} - \beta_4 x_{16}^2)$

$q_5 = (x_{15} - \beta_4 x_{15}^2)$

$q_6 = (x_{12} - \beta_3 x_{12}^2)$

$q_7 = (x_8 - \beta_2 x_8^2)$

$q_8 = (x_4 - \beta_1 x_4^2).$

The upper-level optimization targets maximization of the eight outgoing traffic flows, which are functions of the durations of the previous and next traffic light cycles:

(18) $\max_{c_{i,i=1,\dots,5}} \{q_1(c_1,c_2) + q_2(c_2,c_3) + q_3(c_3,c_4) + q_4(c_4,c_5) + q_5(c_4,c_5) + q_6(c_3,c_4) + q_7(c_2,c_3) + q_8(c_1,c_2)\}$

By substituting (16) – (17) in the goal function (18), it follows

(19) $\max_{c_{i,i=1,\dots,5}} \{(x_5 - \beta_1 x_5^2) + (x_9 - \beta_2 x_9^2) + (x_{13} - \beta_1 x_{13}^2) + (x_{16} - \beta_4 x_{16}^2) + (x_{15} - \beta_4 x_{15}^2) + (x_{12} - \beta_3 x_{12}^2) + (x_8 - \beta_2 x_8^2) + (x_4 - \beta_1 x_4^2)\}$

subject to the constraints

(20) $x_5 = x_{5_0} - S_1 u_1 + S_3 u_2 + 0.9 S_1 c_1 - 0.9 S_3 c_2$

$x_9 = x_{9_0} - S_3 u_2 + S_5 u_3 + 0.9 S_3 c_2 - 0.9 S_5 c_3$

$x_{13} = x_{13_0} - S_5 u_3 + S_7 u_4 + 0.9 S_5 c_3 - 0.9 S_7 c_4$

$x_{16} = x_{16_0} - S_7 u_4 + S_9 u_5 + 0.9 S_7 c_4 - 0.9 S_9 c_5$

$x_{15} = x_{15_0} - S_9 u_5 + S_7 u_4 - 0.9 S_7 c_4 + 0.9 S_9 c_5$

$x_{12} = x_{12_0} - S_7 u_4 + S_5 u_3 - 0.9 S_5 c_3 + 0.9 S_7 c_4$

$x_8 = x_{5_0} - S_5 u_3 + S_3 u_2 + 0.9 S_5 c_3 - 0.9 S_3 c_2$

$x_4 = x_{4_0} - S_3 u_2 + S_1 u_1 - 0.9 S_1 c_1 + 0.9 S_3 c_2$.

The constraints (20) represent the outgoing flows from the network's junctions.

The goal function (19) can be presented in vector's form:

(21) $\max_{c_{i,i=1,\dots,5}} \{c^T Q_3 c + c^T Q_4 u + u^T Q_5 u + Q_6 u + Q_7 c\}$

where the matrices Q_3 - Q_7 are the following

$$Q_3 = \begin{Vmatrix} -1.62\beta_1 S_1^2 & 1.62\beta_1 S_1 S_3 & 0 & 0 & 0 \\ 1.62\beta_1 S_1 S_3 & -1.62(\beta_1+\beta_2)S_3^2 & 1.62\beta_2 S_3 S_5 & 0 & 0 \\ 0 & 1.62\beta_2 S_3 S_5 & -1.62(\beta_2+\beta_3)S_5^2 & 1.62\beta_3 S_5 S_7 & 0 \\ 0 & 0 & 1.62\beta_3 S_5 S_7 & -1.62(\beta_3+\beta_4)S_7^2 & 1.62\beta_4 S_7 S_9 \\ 0 & 0 & 0 & 1.62\beta_4 S_7 S_9 & -1.62\beta_4 S_9^2 \end{Vmatrix}$$

$$Q_4 = \begin{Vmatrix} 3.6\beta_1 S_1^2 & -3.6\beta_1 S_1 S_3 & 0 & 0 & 0 \\ -3.6\beta_1 S_1 S_3 & 3.6(\beta_1+\beta_2)S_3^2 & -3.6\beta_2 S_3 S_5 & 0 & 0 \\ 0 & -3.6\beta_2 S_3 S_5 & 3.6(\beta_2+\beta_3)S_5^2 & -3.6\beta_3 S_5 S_7 & 0 \\ 0 & 0 & -3.6\beta_3 S_5 S_7 & 3.6(\beta_3+\beta_4)S_7^2 & -3.6\beta_4 S_7 S_9 \\ 0 & 0 & 0 & -3.6\beta_4 S_7 S_9 & 3.6\beta_4 S_9^2 \end{Vmatrix}$$

$$Q_5 = \left\| \begin{matrix} -\beta_1 S_1^2 & 2\beta_1 S_1 S_3 & 0 & 0 & 0 \\ 2\beta_1 S_1 S_3 & -2(\beta_1+\beta_2) S_3^2 & 2\beta_2 S_3 S_5 & 0 & 0 \\ 0 & 2\beta_2 S_3 S_5 & -2(\beta_2+\beta_3) S_5^2 & 2\beta_3 S_5 S_7 & 0 \\ 0 & 0 & 2\beta_3 S_5 S_7 & -2(\beta_3+\beta_4) S_7^2 & 2\beta_4 S_7 S_9 \\ 0 & 0 & 0 & 2\beta_4 S_7 S_9 & -2\beta_4 S_9^2 \end{matrix} \right\|$$

$$Q_6 = \left\| \begin{matrix} 2\beta_1(x_{5_0} - x_{4_0})S_1 & 2[\beta_1(x_{4_0} - x_{5_0}) + \beta_2(x_{9_0} - x_{8_0})]S_3 & 2[\beta_2(x_{8_0} - x_{9_0}) + \beta_3(x_{13_0} - x_{12_0})]S_5 \\ & 2[\beta_3(x_{12_0} - x_{13_0}) + \beta_4(x_{16_0} - x_{15_0})]S_7 & 2\beta_4(x_{15_0} - x_{16_0})S_9 \end{matrix} \right\|$$

$$Q_7 = \left\| \begin{matrix} 1.8\beta_1(x_{4_0} - x_{5_0})S_1 & 1.8[\beta_1(x_{5_0} - x_{4_0}) + \beta_2(x_{8_0} - x_{9_0})]S_3 & 1.8[\beta_2(x_{9_0} - x_{8_0}) + \beta_3(x_{12_0} - x_{13_0})]S_5 \\ & 1.8[\beta_3(x_{13_0} - x_{12_0}) + \beta_4(x_{15_0} - x_{16_0})]S_7 & 1.8\beta_4(x_{16_0} - x_{15_0})S_9 \end{matrix} \right\|$$

The upper level optimization problem, formalized by (19) - (20) aims optimal determining of the traffic lights cycles of the all five traffic lights of the urban network. In this problem the values of x_i, $i = 1, \ldots, 18$ and u_i, $i = 1, \ldots, 5$ are received as optimal solutions from the lower level optimization problem (3) - (4). The upper level's optimization problem finds as optimal solution the duration of the traffic light cycles of the five traffic lights. These values of c_i, $i = 1, \ldots, 5$ are sent to the lower level where they become parameters of the lower-level optimization problem (3) - (4). The iterative procedures continue till establishing convergence of the solutions.

4. Simulation and Numerical Results

The simulation of the bi-level optimization is in MATLAB environment using real data for the traffic's network parameters. These data are collected during the working days of a week. The MATLAB's application tool YALMIP is used [17] and the "solve-bilevel" function is called simultaneously for calculations of the lower and upper level optimization problems. The results of the bi-level optimization (named "bilevel" – blue solid line in the figures below) are compared with quadratic optimization problems (dashed black line), solved independently for the lower and upper level like classical optimization problems (*figure 2 - figure 15*).

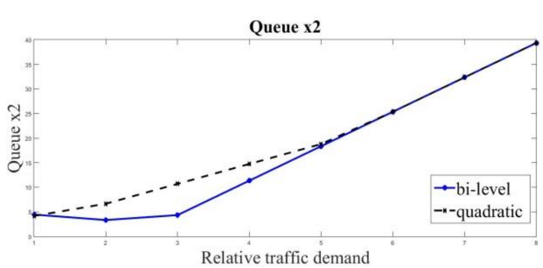

Figure 2. Variation of traffic queue x_2

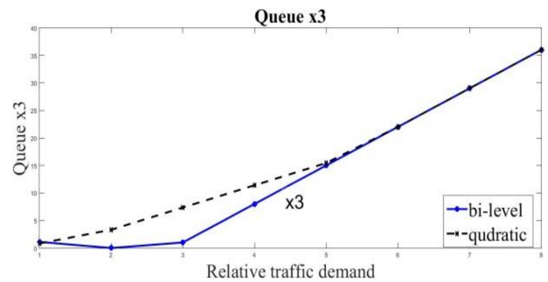

Figure 3. Variation of traffic queue x_3

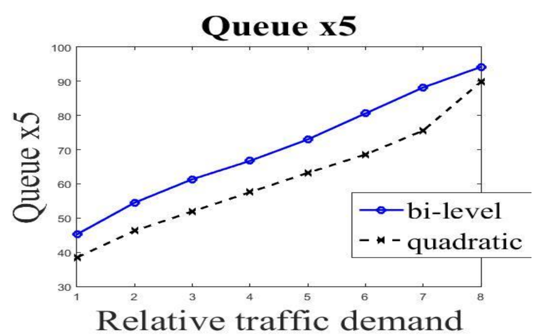

Figure 4. Variation of traffic queue x_5

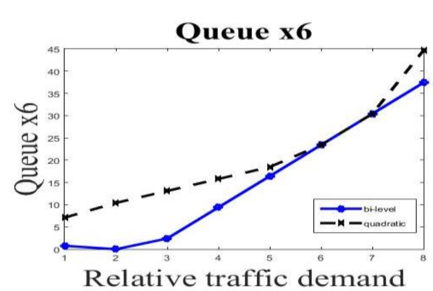

Figure 5. Variation of traffic queue x_6

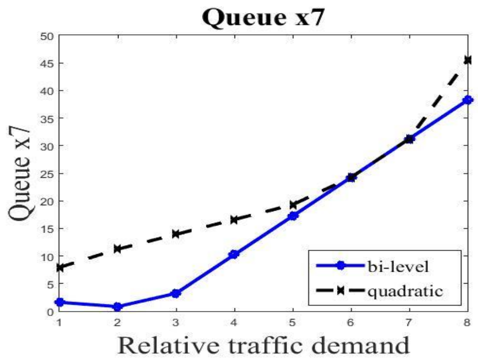

Figure 6. Variation of traffic queue x_7

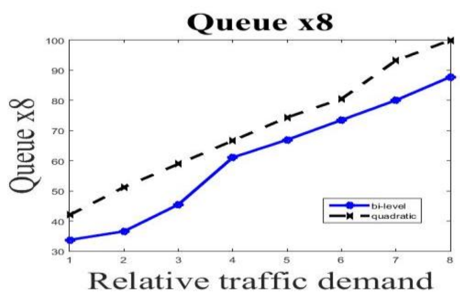

Figure 7. Variation of traffic queue x_8

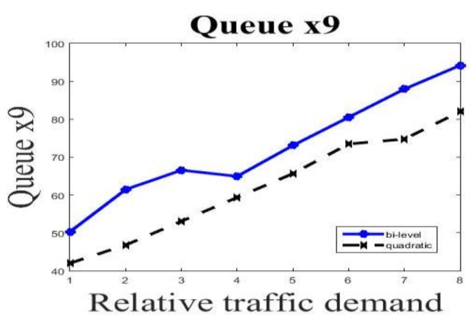

Figure 8. Variation of traffic queue x_9

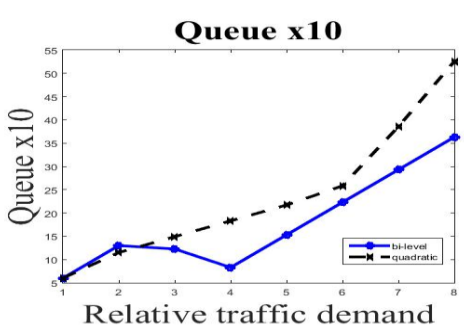

Figure 9. Variation of traffic queue x_{10}

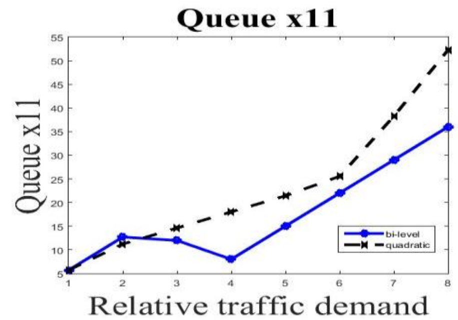

Figure 10. Variation of traffic queue x_{11}

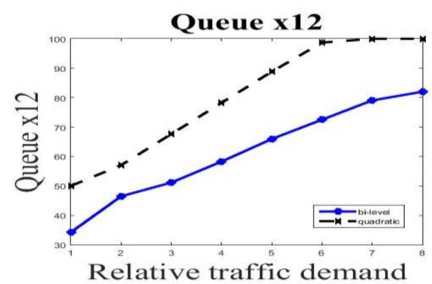

Figure 11. Variation of traffic queue x_{12}

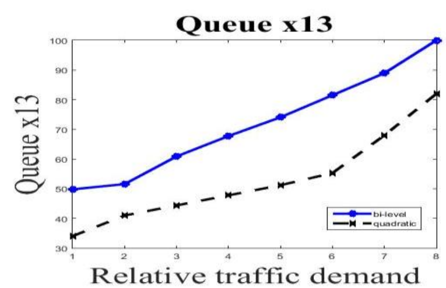

Figure 12. Variation of traffic queue x_{13}

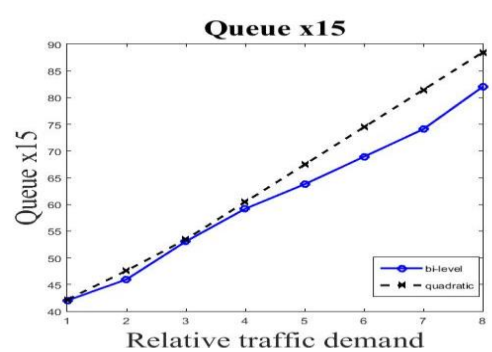

Figure 13. Variation of traffic queue x_{15}

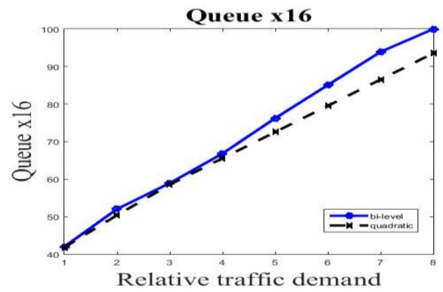

Figure 14. Variation of traffic queue x_{16}

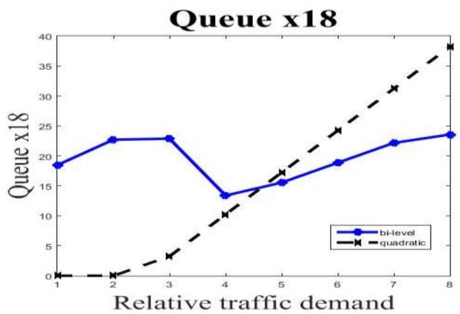

Figure 15. Variation of traffic queue x_{18}

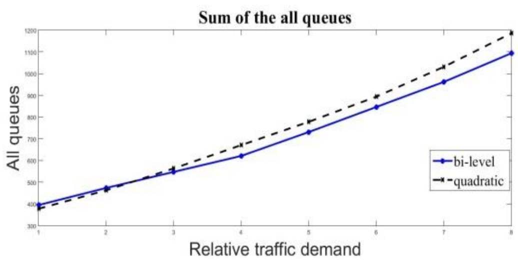

Figure 16. Sum of all traffic queues

The traffic queues of the main direction West to East of *figure 1* (x_5, x_9, x_{13}, x_{16}) are a little bit bigger than by applying quadratic optimization. However, the queue lengths of the opposite main direction – from East to West (x_8, x_{12}, x_{15}, x_{18}) have less values in comparison with the quadratic optimization. Because the queue lengths after bi-level optimization of the perpendicular directions (x_2, x_3, x_6, x_7, x_{10}, x_{11}) are also smaller, we can conclude that the bi-level optimization leads to better results. This is confirmed by the sum of all traffic queues (*figure 16*) which shows lower level of the queues after applying bi-level optimization.

The variation of the green lights durations are presented in *figures 17 - 20*.

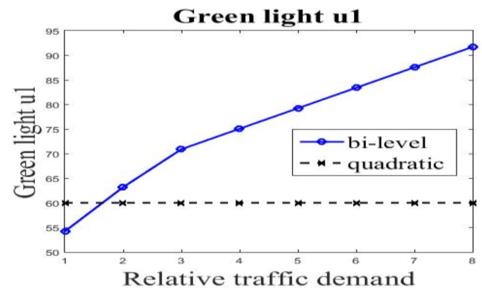

Figure 17. Variation of the green light duration u_1

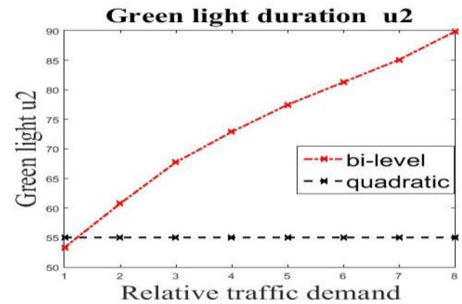

Figure 18. Variation of the green light duration u_2

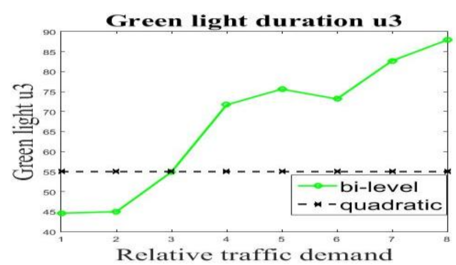

Figure 19. Variation of the green light duration u_3

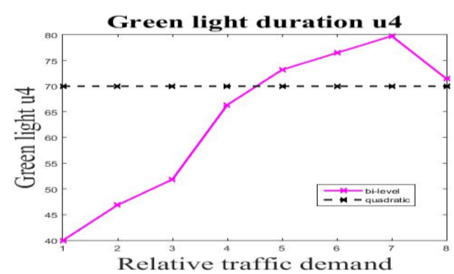

Figure 20. Variation of the green light duration u_4

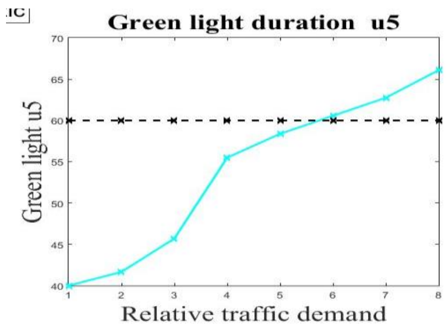

Figure 21. Variation of the green light duration u_5

Because the current case is constant value of the green light duration, in *figures 17 - 21* their values are constant. The solutions of the lower level optimization problem are the queue lengths and the green light durations. It is seen in *figures 17 - 21* the different optimal solutions of the green light durations for the all five crossroad sections.

The next experiments represent comparisons between the bi-level optimization and solving classical quadratic optimization problem with goal function green light durations. The solutions of these problems are given in *figure 22.*

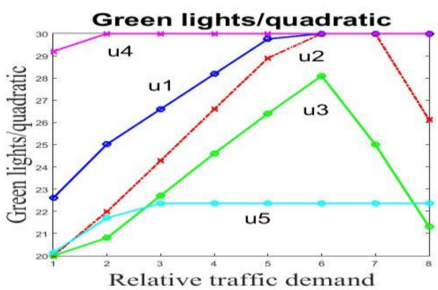

Figure 22. Relative traffic demand by quadratic optimization

Figure 23 illustrates the variation of the green light durations of the all five crossroad sections after bi-level optimization.

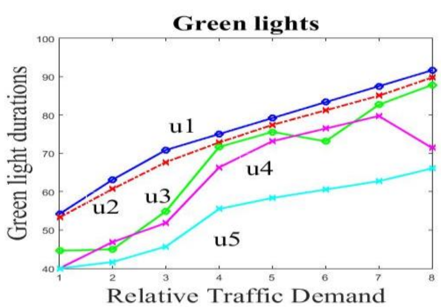

Figure 23. Relative traffic demand by bi-level optimization

Figures 24 - 28 illustrate the dynamics of the green light durations for each of the five traffic lights. In blue solid line is the green light duration as solution of bi-level optimization and the dashed black line is the green light duration applying quadratic optimization.

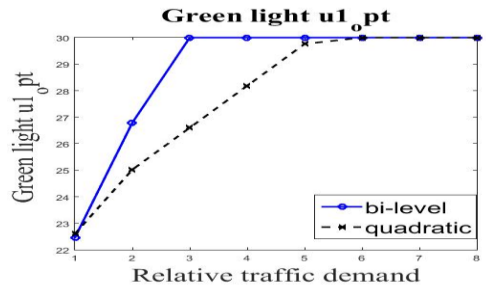

Figure 24. Relative traffic demand for u_1

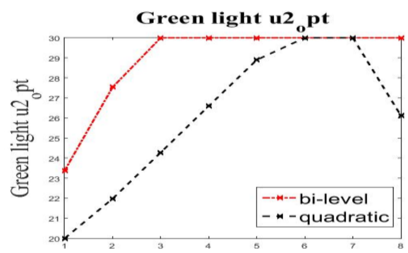

Figure 25. Relative traffic demand for u_2

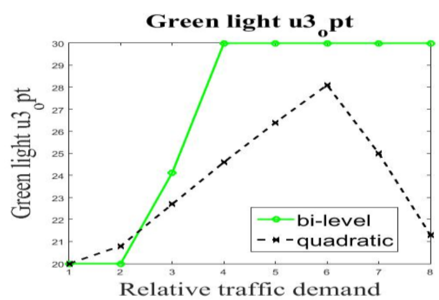

Figure 26. Relative traffic demand for u_3

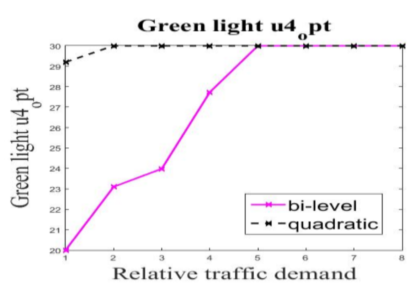

Figure 27. Relative traffic demand for u_4

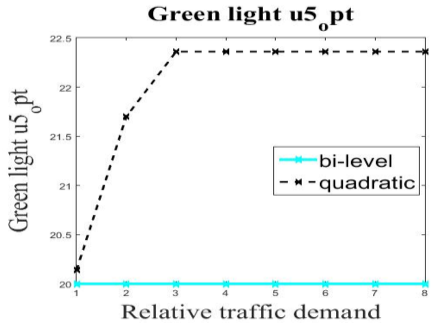

Figure 28. Relative traffic demand for u_5

The difference between the green lights dynsmics can be explained with the different optimization problems. In the classical optimization problem for finding the green light duration only u_i, $i = 1, \dots, 5$ are considered as arguments. The solutions of the bi-level lower problem, which finds the optimal u_i, $i = 1, \dots, 5$, except u_i, $i = 1, \dots, 5$ are taking into account the queue lengths in front of the junctions and the traffic light cycles as optimal solutions from the upper level. These interconnections lead to integration of more arguments, constraints and goals, which result in improving the traffic behavior.

5. Conclusion

This research presents the application of bi-level optimization for improving traffic flow in urban area. Formalization of the lower- and upper-level subproblems is presented. The usage of the hierarchical optimization is caused by its positive advantages like increasing the set of optimization parameters, constraints and goal functions. Instead of optimization of only one goal function with set of parameters and constraints, the bi-level optimization allows optimization of two goal functions, with wider set of arguments and constraints, which leads to better results. The lower level optimization goal is minimization of the queue lengths in front of the junctions. The upper level optimization problem targets maximization of the outgoing from the junction traffic flow by optimizing the duration of the traffic lights cycle. The calculated solutions of each iteration are sent like parameters to the other optimization level. In that manner is realized interconnection between the both optimization problems which obtain two optimal solutions of the both optimization problems, satisfying the constraints of the both optimization problems. In that manner by solving interacted simpler optimization problems is solved complex optimization problem with larger sets of goals, constraints and parameters. A real urban network is considered with traffic data, collected by one week measurements in working days. The simulation results are compared on two folds: with the current state without optimization and with classical optimization problems where the goal and the sets of parameters and constraints are less in comparison with the bi-level optimization. The received results illustrate the improvement of the traffic behavior when bi-level optimization is applied.

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