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Modeling of the 2D Alternating Freezing and Thawing of Logs at Ambient Air Temperature

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Key Words: *2D mathematical models; beech logs; freezing; thawing; icing degree; ambient air temperature.*

Abstract. *This paper presents a methodology for mathematical modeling and research of two mutually connected problems: 2D non-stationary temperature distribution in logs subjected to many days and nights alternating freezing and thawing at periodically changing air temperature near them and change in the icing degree of the logs during these processes. Mathematical descriptions of the periodically changing ambient air temperature and of the icing degree of the logs under influence of that temperature have been carried out. These descriptions are introduced in our mutually connected 2D non-linear mathematical models of the 2D temperature distribution in logs during their freezing and thawing at convective boundary conditions. The paper presents solutions of the models with explicit form of the finite-difference method in the calculation environment of Visual FORTRAN. Results from a simulative investigation of 2D non-stationary temperature distribution and icing degree of beech logs with a diameter of 0.24 m, length of 0.48 m, moisture content of 0.6 kg·kg^{-1}, and initial temperature of 0 °C during their 5 days and nights alternating freezing and thawing at sinusoidal change of the air temperature with various initial values below –5 °C and different amplitudes are presented, visualized, and analyzed.*

1. Introduction

In the accessible specialized literature there are reports about the temperature distribution in subjected to thawing frozen logs only at conductive boundary conditions [2, 4, 6, 7, 12, 13, 18-20, 24-27]. Calculations of the thawing process of logs by heating them in agitated water or steam have been carried out in these publications taking into account that according to [1, 2, 21] the frozen free water in them melts between –2 °C and –1 °C and the frozen bound water melts at temperatures lower than –2 °C.

For different engineering calculations it is needed to be able to determine the icing degree of the wood materials depending on the temperature of the influencing on them air medium and on the duration of their staying in this medium. Such calculations could be carried out using mathematical models, which describe adequately the complex processes of the freezing of both the free and bound water in the wood. The computer solutions of such models can give the non-stationary distribution of the temperature in the materials during their cooling below temperatures, at which a freezing of the whole amount of the free water and a freezing of the depending on the temperature, part of the bound water in the wood occurs.

The wide experimental study of the freezing and thawing processes of logs from different wood species and with various moisture contents above the hygroscopic range, which has been carried out in [10, 30] has shown that the freezing of the free water in the wood occurs in the range between 273.15 K and 272.15 K (i.e., between 0 °C and –1 °C) and after that from T = 272.15 K (i.e., from –1 °C) the freezing of the bound water in the wood begins.

The experiments in these investigations have also shown that the melting of the frozen free water in the wood occurs between 272.15 K and 273.15 K (i.e., between –1 °C and 0 °C). Before that the gradual melting of the frozen bound water in the wood ends at T = 272.15 K (i.e., at –1 °C). All experiments have been carried out at a curvilinear change in the temperature of the air processing medium as follows: in a closed freezer until reaching of a temperature of approximately –30 °C by the logs and after that in an open freezer until the logs reach room temperature.

Mutually connected 2D mathematical models of the transient non-linear heat conduction in logs during their freezing and subsequent thawing in an air environment have been created and solved in [11, 30]. Using the experimentally obtained results mentioned above, the models have been verified at the same curvilinear convective boundary conditions as during the experiments.

The verifying of the models at a curvilinear change of the air processing medium allows us to conduct simulative investigations by the models of the non-stationary temperature distribution and of the icing degree in the logs subjected to the influence of periodic change of the atmospheric temperature in the winter.

The modeling and the multi-parameter study of the mutually connected freezing and thawing processes of logs at atmospheric temperatures are of considerable scientific and practical interest. For example, as a result of such a study it is possible to determine the real icing degree of logs depending on their dimensions, wood species, moisture content, and on the temperature of the air near the logs during their many days staying in an open warehouse before the thermal treatment in the production of veneer. The information about the real value of the icing degree can be used for scientifically based computing and realizing of the optimal, energy saving regimes for thermal treatment of each specific batch of logs.

This paper presents the numerical solving of two mutually connected 2-dimensional mathematical models of the transient non-linear heat conduction in logs during their alternating freezing and thawing at convective boundary conditions with periodically changing atmospheric temperature in the winter. The model of the freezing process takes into account the impact of the internal sources of latent heat of both the free and bound water in the wood on the temperature distribution in the logs [14].

The both models reflect the impact of the temperature on the fiber saturation point of each wood species, with whose participation the current values of the thermo-physical properties in each separate volume point of the subjected to alternating freezing and thawing logs are computed.

The paper also presents and visualizes the results from simulative investigation of the 2D non-stationary temperature distribution and icing degree of beech logs with a diameter of 0.24 m, length of 0.48 m, moisture content of 0.6 kg·kg^{-1}, and initial temperature of 0 °C during their 5 days and nights continuous alternating freezing and thawing at sinusoidal change of the ambient air temperature with different initial values below –5 °C and various amplitudes.

2. Mathematical Model of 2D Temperature Distribution in Logs Subjected to Freezing in Air Medium

When the length of the logs, *L*, is larger than their diameter, *D*, by not more than 3 - 4 times, for the calculation of the change in the temperature in the longitudinal sections of horizontally situated logs (i.e., along the coordinates *r* and *z* of these sections) during their freezing in air medium the following 2D mathematical model can be used [11, 14, 30]:

$$(1)\quad c_{\text{we-fr}}\rho_{\text{w}}\frac{\partial T_{\text{w}}(r,z,\tau)}{\partial \tau}=\text{div}(\lambda_{\text{w-fr}}\ \text{grad}\ T_{\text{w}})+q_{\text{v}}$$

with an initial condition

$$(2)\quad T(r,z,0)=T_{\text{w0}}$$

and boundary conditions for convective heat transfer:

• Along the radial coordinate *r* on the logs' frontal surface during the freezing process:

$$(3)\quad \frac{\partial T(r,0,\tau)}{\partial r}=-\frac{\alpha_{\text{wp-fr}}(r,0,\tau)}{\lambda_{\text{wp}}(r,0,\tau)}\left[T(r,0,\tau)-T_{\text{m-fr}}(\tau)\right];$$

• Along the longitudinal coordinate *z* on the logs' cylindrical surface during the freezing process:

$$(4)\quad \frac{\partial T(0,z,\tau)}{\partial z}=-\frac{\alpha_{\text{wr-fr}}(0,z,\tau)}{\lambda_{\text{wr}}(0,z,\tau)}\left[T(0,z,\tau)-T_{\text{m-fr}}(\tau)\right].$$

Equations (1) - (4) represent a common form of a mathematical model of the freezing process of the logs, i.e., of the 2D temperature distribution in logs subjected to freezing in air medium.

3. Mathematical Model of 2D Temperature Distribution in Logs Subjected to Thawing in Air Medium

When the diameter of the logs, *D*, is smaller than their length, *L*, by not more than 3 - 4 times, for the calculation of the change in *T* in the longitudinal sections of the frozen logs during their thawing in air processing medium the following 2D mathematical model can be used [11, 30]:

$$(5)\quad c_{\text{we-nfr}}\rho_{\text{w}}\frac{\partial T_{\text{w}}(r,z,\tau)}{\partial \tau}=\text{div}(\lambda_{\text{w-nfr}}\ \text{grad}\ T_{\text{w}})$$

with an initial condition

$$(6)\quad T(r,z,0)=T(r,z,\tau_{\text{fre}})$$

and boundary conditions for convective heat transfer:

• Along the radial coordinate r on the logs' frontal surface during the thawing process:

$$(7)\quad \frac{\partial T(r,0,\tau)}{\partial r} = -\frac{\alpha_{\text{wp-thaw}}(r,0,\tau)}{\lambda_{\text{wp}}(r,0,\tau)}\left[T(r,0,\tau) - T_{\text{m-thaw}}(\tau)\right];$$

• Along the longitudinal coordinate z on the logs' cylindrical surface during the thawing process:

$$(8)\quad \frac{\partial T(0,z,\tau)}{\partial z} = -\frac{\alpha_{\text{wr-thaw}}(0,z,\tau)}{\lambda_{\text{wr}}(0,z,\tau)}\left[T(0,z,\tau) - T_{\text{m-thaw}}(\tau)\right].$$

Equations (5) - (8) represent a mathematical model of the logs' thawing process in air medium, i.e., of the 2D temperature distribution in logs subjected to thawing immediately after their freezing.

4. Mathematical Description of the Changing Ambient Air Temperature

For the numerical solving of the given above mutually connected mathematical models of the logs' freezing and thawing processes it is needed to have a mathematical description of the temperature of the air medium in the surrounding environment near the logs, $T_{\text{m-fr}}$ and $T_{\text{m-thaw}}$. The periodic change of the atmospheric temperature T_{m} during the time at a constant value of its amplitude T_{ma} can be described by the following equation [3]:

$$(9)\quad T_{\text{m}} = T_{\text{m0}} + (T_{\text{ma}} - T_{\text{m0}})\sin(\omega\tau),$$

where T_{m0} is the initial value of T_{m}, K; T_{ma} – amplitude value of T_{m}, K; ω – angular frequency of T_{m}, s^{-1}; τ – time, s.

The angular frequency of T_{m} in Eq. (9) is equal to

$$(10)\quad \omega = \frac{2\pi}{\tau_0},$$

where τ_0 is the period of change in T_{m}, s. For the precise solving of tasks with the participation of Eq. (10) it is needed to use $\pi = 3.14159$.

For a periodic change of the air temperature during one day and night, i.e., at $\tau_0 = 1$, d = 24, h = 86,400 s, according to Eq. (10) it is obtained that

$$\omega = \frac{2\pi}{\tau_0} = \frac{2\cdot 3.14159}{86400} = 7.2722\cdot 10^{-5}\ \text{s}^{-1}.$$

A gradually increase or decrease of T_{m0} in the beginning of each subsequent period of T_{m} compared to its initial value, $T_{\text{m0-in}}$, at constant value of the amplitude T_{ma} can be described by the equation:

$$(11)\quad T_{\text{m}} = T_{\text{m0-in}}(1 \pm K_{\text{m0}}\tau) + (T_{\text{ma}} - T_{\text{m0-in}})\sin(\omega\tau),$$

where K_{m0} is a coefficient equal to

$$(12)\quad K_{\text{m0}} = \frac{\dfrac{\Delta T_{\text{m0-}\tau_0}}{T_{\text{m0-in}}}}{\tau_0}$$

and $\Delta T_{\text{m0}-\tau_0}$ is the change in T_{m0} during the time interval equal to τ_0, K; $T_{\text{m0-in}}$ – initial value of the periodically changing temperature T_{m}, K.

If for example $T_{\text{m0-in}}$ is equal to 268.15 K and that value changes by $\Delta T_{\text{m0}-\tau_0}$ = 2 K during each period of τ_0 = 1, d = 86,400 s, then according to Eq. (12) it follows that

$$K_{\text{m0}} = \frac{\dfrac{2}{268.15}}{86400} = 8.6325\cdot 10^{-8}\ \text{s}^{-1}.$$

When the amplitude of T_{m} gradually increases or decreases during the time compared to its initial value, $T_{\text{ma-in}}$, then the temperature T_{m} can be calculated according to the equation

$$(13)\quad T_{\text{m}} = T_{\text{m0}} + [(T_{\text{ma-in}} - T_{\text{m0}})(1 \pm K_{\text{ma}}\tau)]\sin(\omega\tau),$$

where K_{ma} is a coefficient equal to

$$(14)\quad K_{\text{ma}} = \frac{\dfrac{\Delta T_{\text{ma-}\tau_0}}{T_{\text{ma-in}} - T_{\text{m0-in}}}}{\tau_0}$$

and $\Delta T_{\text{ma}-\tau_0}$ is the change in T_{ma} during one period of τ_0, K; $T_{\text{ma-in}}$ – initial value of the amplitude, K.

If for example the initial value of $T_{\text{ma-in}}$ is equal to 293.15 K and that value changes by $\Delta T_{\text{ma}-\tau_0}$ = 2 K during each period of τ_0 = 1, d = 86,400 s, then according to Eq. (14) it follows that

$$K_{\text{ma}} = \frac{\dfrac{2}{288.15 - 268.15}}{86400} = 1.15741\cdot 10^{-6}\ \text{s}^{-1}.$$

The simultaneous change of the values of T_{m0} and T_{ma} in the beginning of each subsequent period compared to their initial values, $T_{\text{m0-in}}$ and $T_{\text{ma-in}}$, respectively, can be described by the equation

$$(15)\quad \begin{aligned} T_{\text{m}} &= T_{\text{m0-in}}(1 \pm K_{\text{m0}}\tau) + \\ &+[(T_{\text{ma-in}} - T_{\text{m0-in}})(1 \pm K_{\text{ma}}\tau)]\sin(\omega\tau), \end{aligned}$$

where the coefficients K_{m0} and K_{ma} are calculated according to Eqs. (12) and (14), respectively.

The signs "+" and "–" in the right-hand side of Eq. (15) are used when the values of $T_{m0\text{-}in}$ or $T_{ma\text{-}in}$ increase or decrease respectively during the periodic change in T_m.

At $K_{m0} = 0$ and $K_{ma} = 0$ Eq. (15) becomes identical to Eq. (9).

5. Mathematical Description of the Thermo-physical Properties and Icing Degree of Logs

Mathematical descriptions of the specific heat capacities of the non-frozen and frozen wood, $c_{\mathrm{w-nfr}}$ and $c_{\mathrm{w-fr}}$, respectively; of the frozen free and the frozen maximal possible amount of bound water in the wood, c_{fw} and c_{bwm}, respectively, and also of the thermal conductivities of non-frozen, $\lambda_{\mathrm{w\text{-}nfr}}$, and frozen wood, $\lambda_{\mathrm{w\text{-}fr}}$, have been suggested in [5-8] based on the experimentally determined in the dissertations [1, 17] data for their change as a function of t and u.

These relations are used in both the European [2, 9, 17, 16, 23, 24, 28, 31] and the American specialized literature [18-20, 25-27] when calculating various processes of wood thermal treatment.

Mathematical descriptions of the effective specific heat capacities of the logs during their freezing and thawing, $c_{\mathrm{we\text{-}fr}}$ and $c_{\mathrm{we\text{-}thaw}}$ respectively, which participate in Eqs. (1) and (5), have been given in [11, 14, 29, 30].

Mathematical descriptions of the wood density above the hygroscopic range, ρ_w, participating in Eqs. (1) and (5), and also of the radial and longitudinal heat transfer coefficients of the logs, $\alpha_{\mathrm{wr\text{-}fr}}$, $\alpha_{\mathrm{wr\text{-}thaw}}$, $\alpha_{\mathrm{wp\text{-}fr}}$, and $\alpha_{\mathrm{wp\text{-}thaw}}$, which participate in the boundary conditions (3), (7), (4) and (8) of the models, have been given in [11, 14, 34].

An approach for computing the values of the icing degrees of logs caused by the freezing separately of the free water, $\Psi^n_{\mathrm{ice-fw}}$, and bound water, $\psi^n_{\mathrm{ice\text{-}bwm\text{-}avg}}$, in them, and also for the calculation of the total icing degree of logs, $\Psi^n_{\mathrm{ice-total}}$ using the already determined current values of $\Psi^n_{\mathrm{ice-fw}}$ and $\psi^n_{\mathrm{ice\text{-}bwm\text{-}avg}}$, is given in [14].

6. Computation of the 2D Temperature Distribution in Logs during their Alternating Freezing and Thawing

The mathematical descriptions of the thermo-physical properties and icing degrees of the logs considered above, and also of the periodically changing atmospheric temperature in winter were introduced in the mutually connected mathematical models (1) - (4) and (5) - (8).

For the numerical solving of the mathematical models, a software program was prepared in the calculation environment of Visual FORTRAN Professional developed by Microsoft. With the help of the program, computations were made for the determination of the 2D non-stationary change of the temperature in the longitudinal sections of three beech logs named below as Log 1, Log 2, and Log 3.

The logs were with a diameter D = 240 mm, length L = 480 mm, basic density $\rho_b = 560$ kg·m^{-3}, moisture content u = 0.6 kg·kg^{-1}, and initial temperature t_{w0} = 0 °C. Two options of 120 h (i.e., of 5 d) continuous alternating freezing and thawing of the logs have been studied as follows:

- For Log 1: at constant values of t_{m0} = –5 °C and t_{ma} = 20 °C;

- For Log 2: at gradual decreasing of the values of $t_{m0\text{-}in}$ = –5 °C and $t_{ma\text{-}in}$ = 20 °C by 2 °C/d;

- For Log 3: at gradual increasing of the value of $t_{m0\text{-}in}$ = –5 °C by 1 °C/d and gradual decreasing of the value of $t_{ma\text{-}in}$ = 20 °C by 2 °C/d.

During the solving of the models, the mathematical description of the thermo-physical properties of beech wood with fiber saturation point $u^{293.15}_{\mathrm{fsp}} = 0.31$ kg·kg^{-1} was used [9, 22]. According to Eq. (12) in [14], at that value of $u^{293.15}_{\mathrm{fsp}}$ the studied logs contain a maximum possible bound water, equal to $u^{272.15}_{\mathrm{fsp}} = 0.331$ kg·kg^{-1}. This means that at accepted u = 0.6 kg·kg^{-1} the logs contain a free water, equal to $u - u^{272.15}_{\mathrm{fsp}} = 0.6 - 0.331 = 0.269$ kg·kg^{-1}.

The models have been solved with the help of explicit schemes of the finite difference method in a way, analogous to the one used and described in [5, 6, 8, 9, 30]. For this purpose, the calculation mesh has been built on ¼ of the longitudinal section of the logs due to the circumstance that this ¼ is mirror symmetrical towards the remaining ¾ of the same section.

The models were solved with a step $\Delta r = \Delta z = 0.006$ m along the coordinates r and z. This means that the number of the steps along r was 20 and along z it was 40, i.e. the total number of the knots in the logs' longitudinal section was equal to $N_{total} = 20 \times 40 = 800$. The interval between the time levels, $\Delta\tau$, (i.e., the value of the step along the time coordinate during the solving of the models), has been determined by the software according to the condition of stability for explicit schemes of the finite difference method and in our case it was equal to 6 s.

On *figure 1* the coordinates of 4 representative points in the logs are given, in which the calculated change in the temperature t was registered and graphically presented. Point 1 is with $r = 30$ mm and $z = 120$ mm; Point 2: with $r = 60$ mm and $z = 120$ mm; Point 3: with $r = 90$ mm and $z = 180$ mm and Point 4: with $r = 120$ mm and $z = 240$ mm (center of the log). These coordinates of the representative points allow for the determination and analyzing of the 2D temperature distribution in logs during their alternating freezing and thawing.

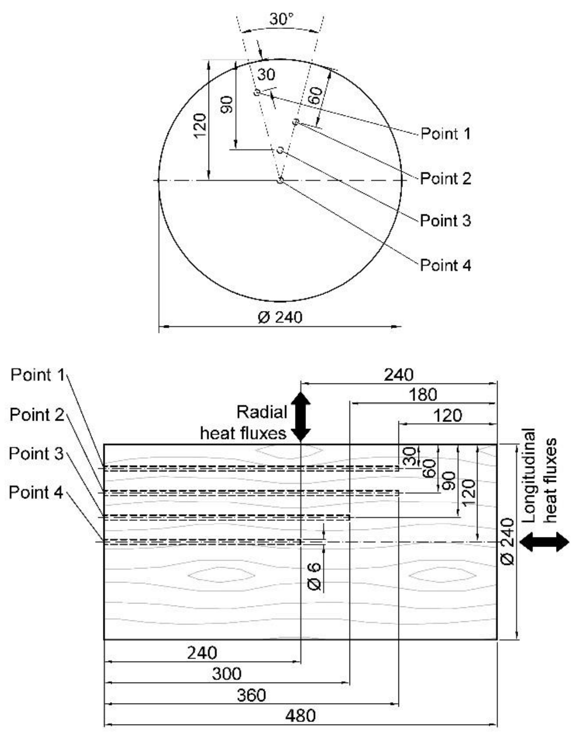

Figure 1. Radial (above) and longitudinal (below) coordinates of 4 representative points of the change in t of logs subjected to alternating freezing and thawing

Figures 2, 3 and *4* show the calculated change in temperature of the processing air medium, t_m, log's surface and average mass temperature, t_s and t_{avg} respectively, and also t of 4 representative points in the studied logs Log 1, Log 2, and Log 3 during their alternating freezing and thawing at mentioned above changes in t_{m0} and t_{ma}.

On *figure 2* it can be seen that at constant values of t_{m0} and t_{ma} after 72[nd] h, i.e. after the 3[rd] period of t_m, a periodical change in the log's temperature with practically constant amplitudes for the separate points is coming.

As far as the point is distanced from the logs' surfaces that much smaller is the amplitude of the periodic change of the temperature in that point. The amplitudes of t_m and t in the separate points after the 3[rd] period are equal to as follows: $t_{ma} = 20.0$ °C, $t_{sa} = 9.5$ °C, $t_{1a} = 8.2$ °C, $t_{2a} = 7.2$ °C, $t_{3a} = 6.5$ °C, and $t_{4a} = 6.0$ °C.

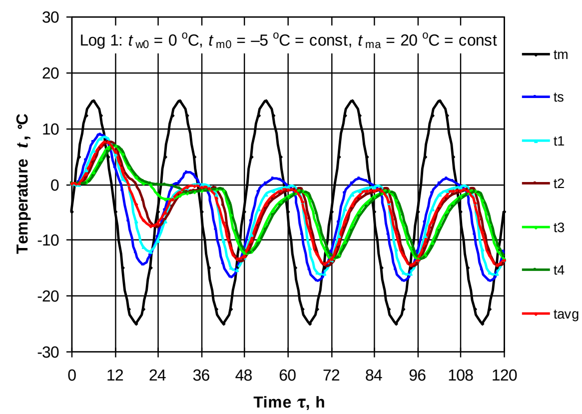

Figure 2. Change in t_m, t_s, t_{avg}, and t of 4 points of the Log 1 during its 120 h alternating freezing and thawing

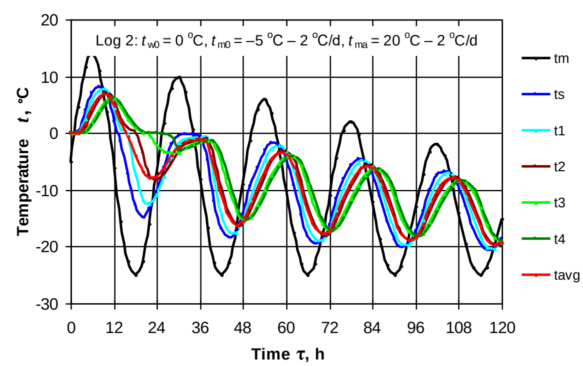

Figure 3. Change in t_m, t_s, t_{avg}, and t of 4 points of the Log 2 during its 120 h alternating freezing and thawing

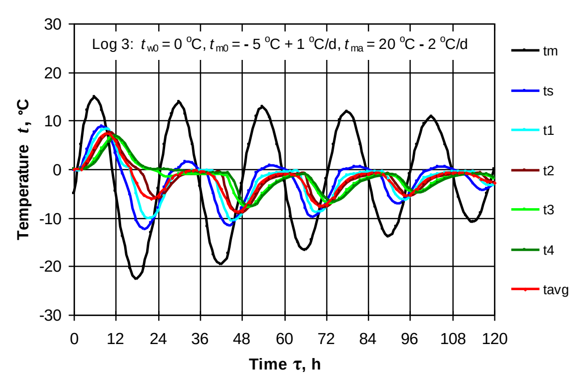

Figure 4. Change in t_m, t_s, t_{avg}, and t of 4 points of the Log 3 during its 120 h alternating freezing and thawing

When t_{m0} and t_{ma} decrease during the time, the amplitudes of t in the separate points also gradually decrease (Fig. 3). At t_{m0} = –5 °C – 2°C/d and t_{ma} = 20 °C – 2 °C/d, during the 5th period t_{ma} = 12.0 °C, t_{sa} = 7.3 °C, t_{1a} = 6.7 °C, t_{2a} = 6.2 °C, t_{3a} = 5.8 °C, and t_{4a} = 5.5 °C. The average mass temperature of the logs, t_{avg}, at 120th h is equal to –13.56 °C for Log 1 and to –19.30 °C for Log 2.

7. Computing the Icing Degree of Logs During their Freezing and Thawing

Synchronously with determination of the 2D non-stationary change of the temperature in the longitudinal sections of the studied logs, calculations of the change in their icing degrees $\Psi^{n}_{\text{ice-fw}}$, $\psi^{n}_{\text{ice-bwm-avg}}$, and $\Psi^{n}_{\text{ice-total}}$ during the alternating freezing and thawing have been carried out.

Figure 5 presents the change in the number of knots of the calculation mesh $N_{\text{ice-fw1}}$ and $N_{\text{ice-fw2}}$ during the 120 h periodically freezing and thawing of the studied logs.

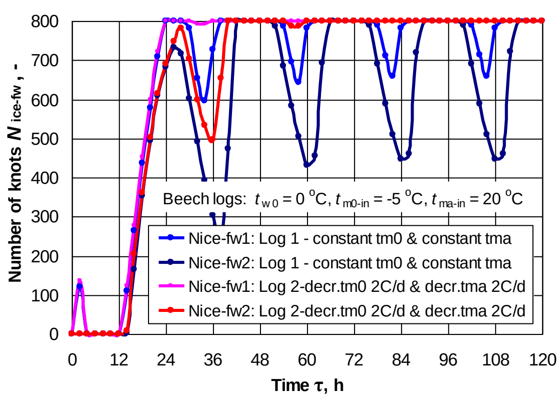

Figure 5. Change in $N_{\text{ice-fw1}}$ and $N_{\text{ice-fw2}}$ during the alternating freezing and thawing of the studied logs

The number of $N_{\text{ice-fw1}}$ is counted when the temperature of each of the knot during the freezing process decreases below 273.15 K (i.e., 0 °C) and a crystallization of the free water in it starts. When during the thawing process the temperature of a given knot increases and reaches 273.15 K, the melting of the frozen free water in that knot ends and the software decreases the current value of $N_{\text{ice-fw1}}$ by 1. The current value of $N_{\text{ice-fw1}}$ is used for the calculation of the icing degree $\Psi_{\text{ice-fw}}$ according to Eq. (9) in (14).

The number of $N_{\text{ice-fw2}}$ is counted when the temperature of each knot during the freezing process decreases below 272.15 K (i.e., –1 °C) and crystallization of the free water in it ends but synchronously with that the crystallization of the bound water starts. When during the thawing process the temperature of a given knot increases and reaches 272.15 K, the melting of the frozen free water in this knot starts and the software decreases by 1 the current value of $N_{\text{ice-fw2}}$.

On *figure 5* it can be seen that during the first 2 hours of the 1st period of t_m the number of $N_{\text{ice-fw1}}$ increases from 0 to 120 due to the circumstance that the temperature of 120 knots in the peripheral layers of the logs then decreases below 273.15 K and a crystallization of the free water in these knots starts.

At the same time the number of $N_{\text{ice-fw2}}$ = 0 because the temperature in all knots of the calculation mesh is higher than 272.15 K. From 2nd to 12th h of the 1st period $N_{\text{ice-fw1}}$ and $N_{\text{ice-fw2}}$ are equal to 0 because the temperature in all knots is higher than 273.15 K.

During the second part of the 1st period of t_m the number of $N_{\text{ice-fw1}}$ increases gradually from 0 to N_{total} = 800. During the next 4 periods $N_{\text{ice-fw1}}$ decreases temporarily for Log 1 from 800 to 595 in 2nd period; to 643 in 3rd period, and to 658 in both the 4th and of 5th periods. At the same time $N_{\text{ice-fw1}}$ for Log 2 decreases from 800 to 782 only during the 2nd period and after that it remains equal to 800 until the end of the 5th period.

During the 2nd, 3rd, 4th, and 5th periods $N_{\text{ice-fw2}}$ decreases temporarily from 800 to the following values:

• 243 in 2nd period, 432 in 3rd period, 447 in both the 4th and in 5th period for Log 1;

• 498 in 2nd period, 786 in 3rd period, and in the whole 4th and in 5th it remains equal to 800 for Log 2.

Figure 6 presents the calculated change in the icing degree of the studied logs, which occurs by the freezing of only the free water in them, $\Psi_{\text{ice-fw}}$. During the first 2 h of the 1st period $\Psi_{\text{ice-fw}}$ increases from 0 to 0.15. This corresponds to the pointed above increasing in $N_{\text{ice-fw1}}$ from 0 to 120. From 2nd to 12th h of the 1st period the icing degree $\Psi_{\text{ice-fw}}$ is equal to 0 because then $N_{\text{ice-fw1}}$ is also equal to 0.

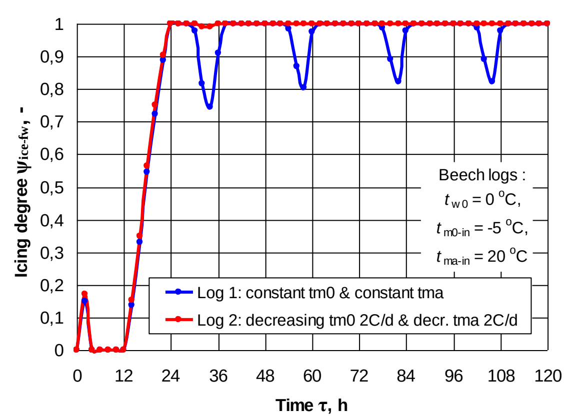

Figure 6. Change in $\Psi_{\text{ice-fw}}$ during the alternating freezing and thawing of the studied logs

During the second part of the 1st period $\Psi_{ice\text{-}fw}$ increases gradually from 0 to $\Psi_{ice\text{-}fw}$ = 1.00. Physically this means that during the 1st period of t_m the whole amount of the free water in the logs crystallizes. After that during the thawing process in the next 4 periods of t_m the degree $\Psi_{ice\text{-}fw}$ of Log 1 decreases temporarily from $\Psi_{ice\text{-}fw}$ = 1.00 and reaches the following values: 0.74 in 2nd period, 0.80 in 3rd period, and 0.82 in both the 4th and 5th periods. For Log 2 a temporary decreasing of $\Psi_{ice\text{-}fw}$ to 0.99 only in 2nd period occurs.

At the end of 120 h alternating freezing and thawing of the studied logs the icing degree $\Psi_{ice\text{-}fw}$ = 1. This means that the whole amount of the free water in the logs, equal to 0.269 kg·kg^{-1} is in a frozen state.

Figure 7 presents the calculated according to Eq. (13) in [14] change in the icing degree of the studied logs, which occurs by the freezing of only the bound water in them, $\Psi_{ice\text{-}bwm\text{-}avg}$. During the first half of the 1st period $\Psi_{ice\text{-}bwm\text{-}avg}$ is equal to 0 for both Log 1 and Log 2 because a freezing of bound water then is absent in them.

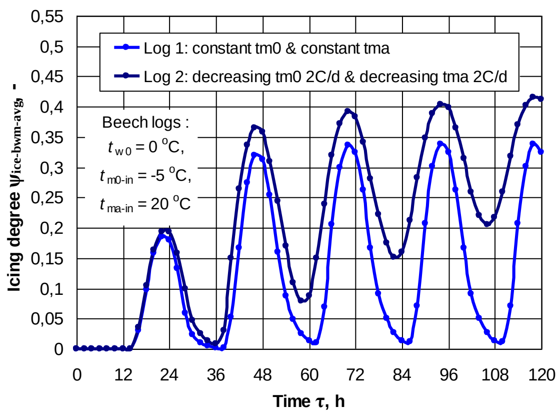

Figure 7. Change in $\Psi_{ice\text{-}bwm\text{-}avg}$ during the alternating freezing and thawing of the studied logs

During the second half of the 1st period $\Psi_{ice\text{-}bwm\text{-}avg}$ increases from 0 to 0.19 for Log 1 and from 0 to 0.20 for Log 2. After that the change in $\Psi_{ice\text{-}bwm\text{-}avg}$ is periodical and during the last 5th period it is in the range from 0.01 to 0.34 for Log 1 and for 0.21 to 0.42 for Log 2.

At the end of 120 h periodic freezing and thawing of the logs the icing degree $\Psi_{ice\text{-}bwm\text{-}avg}$ is equal to 0.3245 for Log 1 and to 0.4115 for Log 2. At the value $u_{fsp}^{272.15}$ = 0.331 kg·kg^{-1} for the beech wood, which was used during the simulations, this means that 0.3245 × 0.331 = 0.107 kg·kg^{-1} of the bound water is in a frozen state and 0.331 – 0.107 = 0.224 kg·kg^{-1} is in a non-frozen state for Log 1 and also that 0.4115 × 0.331 = 0.136 kg·kg^{-1} is in a frozen state and 0.331 – 0.136 = 0.195 kg·kg^{-1} is in a non-frozen state for Log 2.

At the same time, i.e., at the end of the 5th period, the calculated according to Eq. (6) in [14] average mass temperature, T_{avg}, of the studied logs is equal to 259.59 K (i.e., –13.56 °C) for Log 1 and to 253.85 K (i.e., –19.30 °C) for Log 2 (refer to *figures 2* and *3*).

After substitution of these values of T_{avg} in Eq. (18) it is obtained that the amount of the non-frozen water in the wood, u_{nfw}, at the end of the 120th h is equal to 0.224 kg·kg^{-1} for Log 1 and to 0.195 kg·kg^{-1} for Log 2. The equality between the values of u_{nfw}, calculated by Eq. (18) and these determined above with the help of $\Psi_{ice\text{-}bwm\text{-}avg}$ proves the correctness of Eq. (13) in [14] for the calculation of $\Psi_{ice\text{-}bwm\text{-}avg}$.

Figure 8 presents the calculated according to Eq. (25) in [14] change in the total icing degree of the studied logs, which is caused by the freezing of both the free and bound water in them, $\Psi_{ice\text{-}total}$.

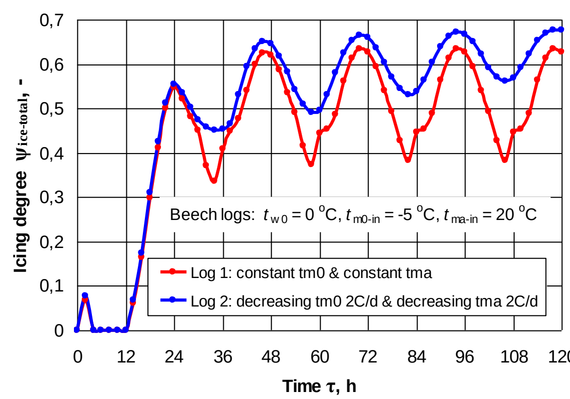

Figure 8. Change in $\Psi_{ice\text{-}total}$ during the alternating freezing and thawing of the studied logs

During the first 2 h of the 1st period of t_m the icing degree $\Psi_{ice\text{-}total}$ increases from 0 to 0.07 due to the freezing of the free water only in some peripheral layers of the logs. From 2nd to 12th h of the 1st period $\Psi_{ice\text{-}total}$ is equal to 0 because both the icing degrees $\Psi_{ice\text{-}fw}$ and $\Psi_{ice\text{-}bwm\text{-}avg}$ then are also equal to 0.

During the second half of the 1st period $\Psi_{ice\text{-}total}$ increases from 0 to 0.55 for Log 1 and from 0 to 0.56 for Log 2. These values of $\Psi_{ice\text{-}total}$ mean that 55% (i.e., 0.33 kg·kg^{-1}) from the whole amount of the moisture content of 0.6 kg·kg^{-1} for Log 1 and 56% (i.e., approximately 0.34 kg·kg^{-1}) for Log 2 are in frozen state.

After the 1st period the change in $\Psi_{ice\text{-}total}$ is periodical and during the last 5th period of t_m it is in the range from 0.38 to 0.64 for Log 1 and for 0.56 to 0.68 for Log 2.

At the end of the 5th period $\Psi_{ice\text{-}total}$ = 0.627 for Log 1 and $\Psi_{ice\text{-}total}$ = 0.675 for Log 2. These values of $\Psi_{ice\text{-}total}$ mean that 62.7% (i.e., 0.376 kg·kg^{-1}) for Log 1 and 67.5% (i.e., 0.405 kg·kg^{-1}) for Log 2 from the whole amount of the moisture content of 0.6 kg·kg^{-1} are then in a frozen state. The rest amounts of u, i.e., 37.3% (i.e., 0.224 kg·kg^{-1}) for

Log 1 and 32.5% (i.e., 0.195 $kg \cdot kg^{-1}$) for Log 2 are in a liquid state in the cell walls of the logs at the end of 120 h alternating freezing and thawing of the studied beech logs.

The equality between the values of u_{nfw}, calculated by Eq. (11) in [14] and these determined with the help of the icing degree $\Psi_{ice\text{-}total}$ proves the correctness of the complex algorithm of 9 steps for the calculation of $\Psi_{ice\text{-}total}$, which has been described in detail in [14].

8. Conclusions

This paper presents a methodology for mathematical modeling and research of two mutually connected problems: 2D non-stationary temperature distribution in logs subjected to many days and nights freezing and thawing at periodically changing air temperature near them in winter and change in the icing degree of the logs during these processes.

Mathematical descriptions of the periodically changing atmospheric temperature in winter and of the icing degree of the logs under influence of that temperature have been carried out.

These descriptions are introduced in our own mutually connected 2D non-linear mathematical models of the 2D temperature distribution in logs during their freezing and thawing.

A software program for the solving of the models and computing of the 2D temperature field and the icing degree of logs has been prepared in FORTRAN, which has been input in the calculation environment of Visual FORTRAN Professional developed by Microsoft.

With the help of the program, computations for the determination of the temperature distribution and icing degrees have been completed for two beech logs with D = 0.24 m, L = 0.48 m, ρ_b = 560 $kg \cdot m^{-3}$, u = 0.6 $kg \cdot kg^{-1}$, $u_{fsp}^{293.15} = 0.31$ $kg \cdot kg^{-1}$, and t_{w0} = 0 °C, subjected to 5 days (i.e., 120 h) continuous alternating freezing and thawing.

Three options of the periodic freezing and thawing of the logs have been studied: at constant values of t_{m0} = –5 °C and t_{ma} = 20 °C for Log 1, at gradual decreasing of these values of $t_{m0\text{-}in}$ and $t_{ma\text{-}in}$ by 2 °C/d for Log 2, and at gradual increasing of $t_{m0\text{-}in}$ by 1 °C/d and gradual decreasing of $t_{ma\text{-}in}$ by 2 °C/d.

It has been determined that at constant values of t_{m0} and t_{ma} after the 72nd h, i.e., after the 3rd period of t_m, a periodical change in the log's temperature with constant amplitudes for the separate points is coming. When $t_{m0\text{-}in}$ and $t_{ma\text{-}in}$ decrease during the time the amplitudes of the temperature in the separate points also gradually decrease.

As far the given point is distanced from the logs' surfaces that smaller the amplitude of the periodically change of the temperature in that point is. It has been calculated that at the end of 120 h alternating freezing and thawing of the studied logs their average mass temperature is equal to –13.56 °C for Log 1 and to –19.30 °C for Log 2. At these values of t_{avg} the wood still contains non-frozen water, equal to 0.224 $kg \cdot kg^{-1}$ for Log 1 and equal to 0.195 $kg \cdot kg^{-1}$ for Log 2.

Three types of the icing degree of logs have been calculated by the models.

The first of them, $\Psi_{ice\text{-}fw}$, is caused by the freezing of only the free water in the wood in the range from 0 °C to –1 °C; the second one, $\Psi_{ice\text{-}bwm\text{-}avg}$, is caused by the freezing of only the bound water in the wood below –1 °C, and the third one, $\Psi_{ice\text{-}total}$, is the total icing degree of the logs, which is a complex mix between $\Psi_{ice\text{-}fw}$ and $\Psi_{ice\text{-}bwm\text{-}avg}$.

It has been computed that after 120 h of the studied processes the icing degrees of the logs reach the following values:

• $\Psi_{ice\text{-}fw}$ = 1, which means that the whole amount of the free water in the studied logs, equal to 0.269 $kg \cdot kg^{-1}$ is in a frozen state;

• $\Psi_{ice\text{-}bwm\text{-}avg}$ = 0.3245 for Log 1 and $\Psi_{ice\text{-}bwm\text{-}avg}$ = 0.4115 for Log 2, which means that from the whole amount of bound water in the logs, equal to 0.331 $kg \cdot kg^{-1}$, in a frozen state are 0.107 $kg \cdot kg^{-1}$ in Log 1 and 0.136 $kg \cdot kg^{-1}$ in Log 2;

• $\Psi_{ice\text{-}total}$ = 0.627 for Log 1 and $\Psi_{ice\text{-}total}$ = 0.675 for Log 2. This means that from the total amount of free and bound water in the logs, equal to 0.6 $kg \cdot kg^{-1}$, in a frozen state are 0.376 $kg \cdot kg^{-1}$ in Log 1 and 0.405 $kg \cdot kg^{-1}$ in Log 2.

The solution of the models allows for the calculation of the temperature distribution and icing degrees, and also various energy characteristics of logs from diverse wood species for each desired moment during their alternating freezing and thawing at periodically changing air temperature with specific parameters.

The results from the solutions of the two mutually connected models with presented above convective boundary conditions can be used for the development of scientifically based energy saving optimized regimes for thermal treatment of frozen logs with consideration of their specific icing degree and also in the software of systems for model predictive automatic control [12, 13, 15] of that treatment.

Symbols

c = specific heat capacity ($J \cdot kg^{-1} \cdot K^{-1}$)
D = diameter (m)
L = length (m)
N = number of knots of the calculation mesh (-)
q = internal latent heat source ($W \cdot m^{-3}$)
R = radius (m): $R = D/2$
r = radial coordinate: $0 \le r \le R$ (m)
S = area (m^2): $S = R \cdot D$
t = temperature (°C): $t = T - 273.15$
T = temperature (K): $T = t + 273.15$
u = moisture content ($kg \cdot kg^{-1}$) = %/100
z = longitudinal coordinate: $0 \le z \le L/2$ (m)
α = heat transfer coefficients between log's surfaces and ambient air medium ($W \cdot m^{-2} \cdot K^{-1}$)
λ = thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)
ρ = density ($kg \cdot m^{-3}$)
τ = time (s)
Δr = step along the coordinates r and z for solving of the models (m)
$\Delta\tau$ = step along the time coordinate for solving of the models (s)
d = day and night: d = 24 h
ψ = relative icing degree (-)
ω = angular frequency (s^{-1})
@ = at
& = and simultaneously with this

Subscripts

a = amplitude
avg = average (for wood mass temperature or for icing degree caused by bound water)
b = basic (for wood density, based on dry mass divided to green volume)
bwm = maximum possible amount of the bound water in the wood
cr = crystallization
fr = freezing
fre = end of freezing
fsp = fiber saturation point
fw = free water
i = knot of the calculation mesh in the direction along the logs' radius: $i = 1, 2, 3, \ldots, 21$
k = knot of the calculation mesh in longitudinal direction of the logs: $k = 1, 2, 3, \ldots, 41$
in = initial
ice = ice
m = medium (for the air near logs during their freezing and thawing)
nfw = non-frozen water
p = parallel to the wood fibers
r = radial direction
s = surface
thaw = thawing
v = volume
w = wood
we = wood effective (for specific heat capacity)
w-fr = wood with frozen water in it
w-nfr = wood with fully liquid water in it
0 = initial or at 0 °C, or a period of the change in the ambient air temperature

Superscripts

n = current number of the step $\Delta\tau$ along the time coordinate for models' solving: $n = 0, 1, 2, \ldots$
272.15 = at 272.15 K, i.e., at –1 °C
293.15 = at 293.15 K, i.e., at 20 °C

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