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Adaptive Tuning Functions Tracking Control with Nonlinear Adaptive Observers

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Abstract. *The paper is dedicated to the derivation of a unified approach for nonlinear adaptive closed loop system design with nonlinear adaptive state and parameter observers combined with tuning functions-based nonlinear adaptive control for trajectory tracking. The proposed approach guarantees asymptotic stability of the closed loop nonlinear adaptive system with respect to the tracking and state estimation errors and Lyapunov stability of the parameter estimator. The advantages of the approach are the lack of over-parametrization, resulting in a minimal number of estimator equations and the preservation of the overdamped performance specifications of the closed loop nonlinear adaptive system in its whole range of operation. The application of the approach to a permanent magnet synchronous motor driven inverted pendulum concludes with simulation of the closed loop nonlinear adaptive system time responses.*

1. Introduction

An overall specific feature of nonlinear and adaptive control is the extensive use of successive nonlinear transformations of the original nonlinear system for nonlinear adaptive trajectory tracking control with nonlinear adaptive state and parameter observers. This is imposed by the lack of necessary and sufficient conditions for stability and asymptotic stability for general nonlinear systems. This fact forces the research of specific classes of nonlinear system models which permit the mentioned tasks to be solved via mathematical tools of known sufficient stability conditions. This results into the use of nonlinear observability and observer canonical forms and transformations [9], transition into an output feedback form [5], nonlinear adaptive observer canonical form (AOCF) [1,2,6] with filtered transformation [8], and methods for nonlinear adaptive control design which use stability conditioned transformations.

A powerful tool for nonlinear state observers design is the state transformation of the original nonlinear system into appropriate nonlinear observability and observer canonical forms [9]. The transformation into a reduced generalized observer canonical form (RGOCF) is achieved by a specific differential splitting approach applied to the generalized characteristic equation of the original nonlinear system. This canonical form is a necessary stage for the transformation into AOCF, used for asymptotically stable nonlinear adaptive state observer design. Generally, the transformation into AOCF depends on the unknown parameters. Hence, the asymptotic state estimation of the original system requires exact parameters estimation. The exact estimation of the parameters is achieved via persistent excitation of the system [1,2,7], but this approach is incompatible with the closed loop system control goals. In this sense, a relevant solution is the use of asymptotically stable adaptive observer with Lyapunov stable parameter estimator [6,7] based on the strict passivity of the AOCF and the Mayer-Kalman-Yakubovic lemma [16]. Classical methods for nonlinear adaptive control [3,4,5,10,11,13,15] are used for trajectory tracking nonlinear adaptive control design, implementing the adaptive estimated state in the feedback control. The adaptive tuning functions control design approach [5], being an enhancement of the adaptive backstepping method, is in fact an iteratively computed specific nonlinear transformation achieving asymptotic stability of the transformed closed loop nonlinear adaptive system.

The paper presents harmonic unification of these approaches simultaneously solving the relevant control tasks for closed loop nonlinear adaptive trajectory tracking control of permanent magnet synchronous motors (PMSM) [12,14] via transformation into RGOCF and AOCF. The nonlinear adaptive parameter and state observers are incorporated in the adaptive tuning functions control, thus avoiding the over-parametrization typical for the classical backstepping approach.

2. Nonlinear Adaptive Observer Design

The original systems considered in our approach are described by the general nonlinear model

(1a) $\dot{\boldsymbol{\eta}} = \mathbf{f}_\eta(\boldsymbol{\eta}, u)$, $\boldsymbol{\eta}(0) = \boldsymbol{\eta}_0$;

(1b) $y = h(\boldsymbol{\eta})$,

where $\boldsymbol{\eta} \in R^n$, $u \in R^1$, $y \in R^1$, $f_\eta(\boldsymbol{\eta}, u) : R^n \times R^1 \to R^n$. It is assumed that the original nonlinear system (1) is transformable in an output feedback canonical form [5]

(2a) $\dot{x} = Ax + f(y) + F(y, u)\theta$, $x(0) = x_0$;

(2b) $y = \boldsymbol{C}x$,

where the matrix pair (**A**, **C**) is in Brunovski canonical form, $\boldsymbol{\theta} \in R^p$ is the unknown parameter vector, $f(y) : R^1 \to R^n$, $\mathbf{F}(y, u) : R^1 \times R^1 \to R^{n\times p}$. The output feedback form is a special case of the reduced generalized observer canonical form [9] for which $\mathbf{F}(y,u)$ has the form

$$\mathbf{F}(y,u)=\left[\mathbf{\Phi}(y),\begin{bmatrix}\mathbf{0}_{(\rho-1)\times(m+1)}\\ \mathbf{I}_{m+1}\end{bmatrix}\sigma(y)u\right],$$

with $\mathbf{\Phi}(y):R^1\to R^{q\times n}$ and $\sigma(y)>0$, where $\rho = n - m$ is the relative degree of the system, m is the zero dynamics order and $\mathbf{\theta} = [\mathbf{\theta}^T_s,\mathbf{\theta}^T_u]^T$ with the parameters $\mathbf{\theta}_u \in R^{m+1}$ in front of the control u and the parameters $\mathbf{\theta}_s \in R^{p-m+1}$, multiplying the output functions. The filtered transformation is used for the design of a nonlinear adaptive observer

(3a) $\mathbf{z}(\mathbf{x}) = \mathbf{x} - \begin{bmatrix}0\\ \xi+\mathbf{\Omega\theta}\end{bmatrix};$

(3b) $\dot{\xi} = A_b\xi + B_b\mathbf{f}(y),\ \xi(\mathbf{0}) = \mathbf{0};$

(3c) $\dot{\mathbf{\Omega}} = A_b\mathbf{\Omega} + B_b\mathbf{F}(y, u),\ \mathbf{\Omega}(\mathbf{0}) = \mathbf{0},$

where the matrices A_b and B_b are given by

$$\mathbf{A}_b=\begin{bmatrix}-\mathbf{b} & \begin{matrix}\mathbf{I}_{(n-2)\times(n-2)}\\ \mathbf{0}_{1\times(n-2)}\end{matrix}\end{bmatrix},\ \mathbf{B}_b=[-\bar{\mathbf{b}},\mathbf{I}_{(n-1)\times(n-1)}],$$

and the vectors $\bar{\mathbf{b}}$ and $\mathbf{b}$ are defined by the coefficients of the $(n-1)$-dimensional Hurwitz polynomial $B(s) = s^{n-1} + b_1s^{n-2} + ... + b_{n-1}$ as

$$\bar{\mathbf{b}}=\begin{bmatrix}b_1\\ \vdots\\ b_{n-1}\end{bmatrix},\ \mathbf{b}=\begin{bmatrix}1\\ \bar{\mathbf{b}}\end{bmatrix}.$$

Applying the transformation (3) for system (2) gives the adaptive observer canonical form

(4a) $\dot{\mathbf{z}} = \mathbf{Az} + \mathbf{b}(\omega_0 + \boldsymbol{\omega}^T\mathbf{\theta}),\ \mathbf{z}(\mathbf{0}) = \mathbf{z}_0;$

(4b) $y = \mathbf{Cz},$

with

(5a) $\omega_0 = \mathbf{\xi}_{(1)} + \mathbf{Cf}(y);$

(5b) $\omega^T = \mathbf{\Omega}_{(1)} + \mathbf{Cf}(y, u),$

where $\mathbf{\xi}_{(1)}$ and $\mathbf{\Omega}_{(1)}$ denote the first rows of the vector ξ and the matrix Ω. For a system with a relative degree $\rho = n$, the filter matrix dynamics (3c) can be split as $\dot{\mathbf{\Omega}} = [\mathbf{\Psi}, \dot{\mathbf{v}}]$ with

(6a) $\dot{\mathbf{\Psi}} = \mathbf{A}_b\mathbf{\Psi} + \mathbf{B}_b\mathbf{\Phi}(y)\ ,\ \mathbf{\Psi}(0) = \mathbf{0};$

(6b) $\dot{\mathbf{v}} = \mathbf{A}_b\mathbf{v} + \mathbf{B}_b\boldsymbol{\varphi}(u)\ ,\ \mathbf{v}(0) = \mathbf{0},$

where $\boldsymbol{\varphi}(u) = e_n\sigma(y)u$ with n-th coordinate vector $e_n = [0, 0, ..., 1]^T$. The nonlinear adaptive observer for the system (4) is chosen as

(7a) $\dot{\hat{\mathbf{z}}} = \mathbf{A}\hat{\mathbf{z}} + \mathbf{b}(\omega_0 + \boldsymbol{\omega}^T\hat{\mathbf{\theta}}) + \mathbf{N}(y-\hat{y})\ ,\ \hat{\mathbf{z}}(0)=\hat{\mathbf{z}}_0\ ;$

(7b) $\hat{y} = \mathbf{C}\hat{\mathbf{z}}\ ,$

with an observer gain matrix $\mathbf{N}$ and parameter estimates $\hat{\mathbf{\theta}}$.

The observer error $\tilde{\mathbf{z}} = \mathbf{z} - \hat{\mathbf{z}}$ is governed by

(8) $\dot{\tilde{\mathbf{z}}} = (\mathbf{A}-\mathbf{NC})\tilde{\mathbf{z}} + \mathbf{b}\boldsymbol{\omega}^T\tilde{\mathbf{\theta}},\quad \tilde{\mathbf{z}}(0) = \tilde{\mathbf{z}}_0$

where the parameter estimates error vector is denoted by $\tilde{\mathbf{\theta}} = \mathbf{\theta} - \hat{\mathbf{\theta}}$. With the proper choice of $\mathbf{N}$ making $(\mathbf{A} - \mathbf{NC})$ stable, the transfer function of system (8)

$$\tilde{\mathbf{y}} = [\mathbf{C}(s\mathbf{I}-(\mathbf{A}-\mathbf{NC}))^{-1}\mathbf{b}]\boldsymbol{\omega}^T\tilde{\mathbf{\theta}} = \frac{s^{n-1}+b_1s^{n-2}+\cdots+b_{n-1}}{s^n+n_1s^{n-1}+n_{n-1}s+\cdots+n_n}\boldsymbol{\omega}^T\tilde{\mathbf{\theta}}$$

is strictly positively real. For such transfer functions the Mayer-Kalman-Yakubovic lemma [16] states that there exists a solution $\mathbf{P} = \mathbf{P}^T > 0$ of Lyapunov equation $(\mathbf{A} - \mathbf{NC})^T\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{NC}) = -\mathbf{Q}$, for which the Hurwitz polynomial vector $\mathbf{b}$ satisfies

(9) $\mathbf{b}^T\mathbf{P} = \mathbf{C}.$

The adaptive parameter estimator dynamics $\dot{\hat{\mathbf{\theta}}}$ is designed via the direct Lyapunov method. A positive definite Lyapunov function candidate is defined as

(10) $V_o = \frac{1}{2}\tilde{\mathbf{z}}^T\mathbf{P}\tilde{\mathbf{z}} + \frac{1}{2}\tilde{\mathbf{\theta}}^T\mathbf{\Gamma}^{-1}\tilde{\mathbf{\theta}},$

whose total time derivative, assuming that the parameters are constant $(\dot{\mathbf{\theta}} = \mathbf{0})$, is

(11) $\dot{V}_o = \tilde{\mathbf{z}}^T[(\mathbf{A}-\mathbf{NC})^T\mathbf{P}+\mathbf{P}(\mathbf{A}-\mathbf{NC})]\tilde{\mathbf{z}} + \tilde{\mathbf{\theta}}^T(\boldsymbol{\omega}\mathbf{b}^T\mathbf{P}\tilde{\mathbf{z}} - \mathbf{\Gamma}^{-1}\dot{\hat{\mathbf{\theta}}}).$

The parameter estimator dynamics

(12) $\dot{\hat{\mathbf{\theta}}} = \mathbf{\Gamma}\boldsymbol{\omega}\quad - y,\ \hat{\mathbf{\theta}}(0) = \hat{\mathbf{\theta}}_0,$

considering (9) reduces the derivative (11) to $\dot{V}_o = -\tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} - \tilde{\mathbf{\theta}}^T(\mathbf{\Gamma}^{-1}\dot{\hat{\mathbf{\theta}}} - \boldsymbol{\tau}_0)$, where the observer tuning function $\boldsymbol{\tau}_0$ is defined as

(13) $\boldsymbol{\tau}_0 = \boldsymbol{\omega}(y-\hat{y}).$

According to the La-Salle-Yoshizawa convergence theorem for non-autonomous nonlinear systems, the observer error $\tilde{\mathbf{z}}$ will be asymptotically stable while the parameter errors $\tilde{\mathbf{\theta}}$ are only Lyapunov stable. This completes the nonlinear adaptive observer design.

3. Adaptive Tuning Functions Tracking Control Design

The control task is the tracking of a reference trajectory $y_r(t), \dot{y}_r(t), \ldots, y_r^{(\rho)}(t)$. To prepare for the adaptive tuning functions design, the output derivative $\dot{y} = \dot{z}_1$ is rewritten in the form

(14) $\dot{y} = \theta_u v_1 + \hat{z}_2 + \omega_0 + \bar{\boldsymbol{\omega}}^T\mathbf{\theta} + \tilde{z}_2$

where $\bar{\boldsymbol{\omega}}^T = [\mathbf{\Psi}_{(1)} + \mathbf{\Phi}_{(1)}, 0]$ and z_2 is replaced by $\hat{z}_2 + \tilde{z}_2$. A natural choice of the virtual control is the variable v_1 as its $(\rho - 1)$th derivative contains the actual control $\dot{v}_{\rho-1} = -b_{\rho-1}v_{\rho-1} + \sigma(y)u$.

In order to turn the tracking problem into a stabilization task, the error coordinates are defined as

(15a) $\varepsilon_1 = y - y_r$,

(15b) $\varepsilon_k = v_{k-1} - \hat{\delta}\dot{y}_r^{(k-1)} - \alpha_{k-1}$, k = 2, ..., ρ,

where $\hat{\delta}$ is the estimate of $\delta = 1 / \theta_u$ and α_{k-1} are the stabilizing functions. It is assumed that the sign of θ_u is known. The tracking error ε_1 dynamics obtained from (15a) and (14) reads $\dot{\varepsilon}_1 = \theta_u v_1 + \hat{z}_2 + \omega_0 + \bar{\boldsymbol{\omega}}^T\boldsymbol{\theta} + \tilde{z}_2 - \dot{y}_r$.

Substituting v_1 from (15b) for $k = 2$ and scaling α_1 as $\alpha_1 = \hat{\delta}\bar{\alpha}_1$, the above equation becomes

(16) $\dot{\varepsilon}_1 = \bar{\alpha}_1 + \hat{z}_2 + \omega_0 + \bar{\boldsymbol{\omega}}^T\boldsymbol{\theta} + \tilde{z}_2 - \theta_u(\dot{y}_r + \bar{\alpha}_1)\tilde{\delta} + \theta_u\varepsilon_2$.

The task is to stabilize (16) with the Lyapunov function candidate

$$V_1 = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2\gamma_\delta}|\theta_u|\tilde{\delta}^2 + V_o,$$

whose total time derivative along the solution of (16) is

(17) $\dot{V}_1 = \varepsilon_1(\alpha_1 + \hat{z}_2 + \omega_0 + \boldsymbol{\omega}^T\boldsymbol{\theta} + \tilde{z}_2 - \theta_u(\dot{y}_r + \alpha_1)\tilde{\delta} + \theta_u\varepsilon_2) - \gamma_\delta^{-1}|\theta_u|\tilde{\delta}\dot{\hat{\delta}} - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} - \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_0)$

The choice of $\bar{\alpha}_1$ as

(18) $\bar{\alpha}_1 = -c_1\varepsilon_1 - d_1\varepsilon_1 - \hat{z}_2 - \bar{\boldsymbol{\omega}}^T\hat{\boldsymbol{\theta}} - \omega_0$,

aims elimination of the known terms involving the tracking error ε_1 in equation (17). The convergence of the error ε_1 is controlled by the coefficient $c_1 > 0$. The term $\tilde{z}_2$ in (17) is treated as a disturbance. That is why the damping term $-d_1\varepsilon_1$ with $d_1 > 0$ is added to suppress its destabilizing effect. With the dependence (18), the Lyapunov function derivative (17) reads

$$\dot{V}_1 = -c_1\varepsilon_1^2 - d_1\varepsilon_1^2 + \varepsilon_1\tilde{z}_2 + \varepsilon_1(\boldsymbol{\omega}^T\tilde{\boldsymbol{\theta}} + \tilde{\theta}_u\varepsilon_2 + \hat{\theta}_u\varepsilon_2 - \theta_u(\dot{y}_r + \alpha_1)\tilde{\delta}) - \gamma_\delta^{-1}|\theta_u|\tilde{\delta}\dot{\hat{\delta}} - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} - \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_0).$$

Before the choice of the parameter estimator dynamics, the following terms are examined

(19) $\boldsymbol{\omega}^T\tilde{\boldsymbol{\theta}} + \theta_u\varepsilon_2 = \boldsymbol{\omega}^T\tilde{\boldsymbol{\theta}} + \tilde{\theta}_u\varepsilon_2 = \boldsymbol{\omega}^T\tilde{\boldsymbol{\theta}} + (v_1 - \hat{\delta}\dot{y}_r - \alpha_1)e_n^T\tilde{\boldsymbol{\theta}} = (\boldsymbol{\omega} - \hat{\delta}(\dot{y}_r + \alpha_1)e_n)^T\tilde{\boldsymbol{\theta}}$,

where, the error ε_2 is substituted from equation (15b). Considering (19) and combining the terms including $\tilde{\boldsymbol{\theta}}$ and $\tilde{\delta}$, the derivative $\dot{V}_1$ becomes

$$\dot{V}_1 = -(c_1 + d_1)\varepsilon_1^2 + \varepsilon_1\tilde{z}_2 + \varepsilon_1\varepsilon_2\hat{\theta}_u - \gamma_\delta^{-1}\tilde{\delta}(|\theta_u|\dot{\hat{\delta}} + \theta_u\gamma_\delta(\dot{y}_r + \alpha)\varepsilon_1) - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} - \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_1),$$

where

(20) $\boldsymbol{\tau}_1 = \boldsymbol{\tau}_0 + (\boldsymbol{\omega} - \hat{\delta}(\dot{y}_r + \alpha)e_n)\varepsilon_1$

Then with the choices

(21) $\dot{\hat{\delta}} = -\gamma_\delta sign(\theta_u)(\dot{y}_r + \alpha)\varepsilon_1, \hat{\delta}(0) = \hat{\delta}_0$,

the above derivative $\dot{V}_1$ reduces to

(22) $\dot{V}_1 = -c_1\varepsilon_1^2 - d_1\varepsilon_1^2 + \varepsilon_1\tilde{z}_2 + \varepsilon_1\varepsilon_2\hat{\theta}_u - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} - \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_1)$.

The parameter estimator dynamics $\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\tau}_1$ will eliminate the error $\tilde{\boldsymbol{\theta}}$ containing a term in $\dot{V}_1$, but this will lead to ρ different parameter estimators dynamics for every step of the approach. By the tuning functions $\boldsymbol{\tau}_k$, $k = 1, 2, ..., \rho$ the choice of $\hat{\boldsymbol{\theta}}$ is postponed until the last step, thus eliminating the overparametrization. This is the advantage of the adaptive tuning functions method over the classical adaptive backstepping method.

The steps of the adaptive tuning functions approach for $k = 2, 3, ..., \rho$ involve deriving the dynamics of ε_k according to (15b). Lyapunov functions candidates are defined recursively as

(23a) $V_k = \frac{1}{2}\varepsilon_k^2 + V_{k-1}$.

The stabilizing function

$$\alpha_k = -\varepsilon_{k-1} - c_k\varepsilon_k - d_k\left(\frac{\partial\alpha_{k-1}}{\partial y}\right)^2\varepsilon_k + b_{k-1}v_1 + \frac{\partial\alpha_{k-1}}{\partial y}(\hat{z}_2 + \omega_0 + \boldsymbol{\omega}^T\hat{\boldsymbol{\theta}}) + \frac{\partial\alpha_{k-1}}{\partial\hat{\mathbf{z}}^T}\dot{\hat{\mathbf{z}}} + \frac{\partial\alpha_{k-1}}{\partial\boldsymbol{\xi}^T}\dot{\boldsymbol{\xi}} + \frac{\partial\alpha_{k-1}}{\partial\boldsymbol{\Psi}}\dot{\boldsymbol{\Psi}} + \frac{\partial\alpha_{k-1}}{\partial\mathbf{v}^T}\dot{\mathbf{v}} + \sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial y_r^{(j-1)}}y_r^{(j)} + \frac{\partial\alpha_{k-1}}{\partial\hat{\boldsymbol{\theta}}^T}\boldsymbol{\Gamma}\boldsymbol{\tau}_k + \left(y_r^{(k-1)} + \frac{\partial\alpha_{k-1}}{\partial\hat{\delta}}\right)\dot{\hat{\delta}} - \sum_{j=2}^{k-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\boldsymbol{\theta}}^T}\boldsymbol{\Gamma}\frac{\partial\alpha_{k-1}}{\partial y}\varepsilon_j,$$

chosen according the Lyapunov function V_k provides the stabilization of the dynamics ε_k where for $k = 2$ the term $-\hat{\theta}_u\varepsilon_1$ is used instead of $-\varepsilon_1$. The tuning functions defined are

$\boldsymbol{\tau}_k = \boldsymbol{\tau}_{k-1} - \frac{\partial\alpha_{k-1}}{\partial y}\boldsymbol{\omega}\varepsilon_k$.

At the final step $k = \rho$ the adaptive control law is chosen as $u = \frac{1}{\sigma(y)}(\alpha_\rho + \hat{\delta}y_r^{(\rho)})$, with parameter estimator dynamics

(24) $\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\tau}_\rho$.

Using the tuning functions approach, the parameter estimates of the nonlinear adaptive observer and the nonlinear adaptive control have unified dynamics (24). In this way the undesirable overparametrization is eliminated.

4. Application of the Approach

The approach will be applied to a permanent magnet synchronous motor (PMSM) driven inverted pendulum system of second order. For this purpose the general approach described in the previous sections will be rendered for order of the system $n = 2$ and relative degree $\rho = n$, presuming lack of zero dynamics. In the previous section the first step of the approach was derived and the results are the tuning function (20) including the observer tuning function (13), the parameter estimator (21), the stabilizing function $\alpha_1 = \hat{\delta}\bar{\alpha}_1$ which according to (18), is $\alpha_1 = -\hat{\delta}(c_1\varepsilon_1 + d_1\varepsilon_1 + \hat{z}_2 + \bar{\boldsymbol{\omega}}^T\hat{\boldsymbol{\theta}} + \omega_0)$, and the Lyapunov function (22). The relations for the second step of the approach are presented as follows. The dynamics of the error ε_2 according to (15b) will be

(25) $\dot{\varepsilon}_2 = -b_1v_1 + \sigma(y)u - \hat{\delta}\ddot{y}_r - \frac{\partial\alpha}{\partial\boldsymbol{\mu}}\dot{\boldsymbol{\mu}} - \frac{\partial\alpha}{\partial y}(\hat{z}_2 + \tilde{z}_2 + \omega_0 + \boldsymbol{\omega}^T\hat{\boldsymbol{\theta}} + \boldsymbol{\omega}^T\tilde{\boldsymbol{\theta}})$

where $\boldsymbol{\mu} = [\hat{\delta}, y_r, \xi_1, \xi_2, \ldots, \xi_n, \boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \ldots, \boldsymbol{\Psi}_{p-1}, v_1, \hat{z}_2, \hat{\boldsymbol{\theta}}^T]^T$ with $\boldsymbol{\Psi}_k$, $k = 1, 2, \ldots, \rho-1$ are the columns of the matrix $\boldsymbol{\Psi}$. Then the respective Lyapunov function candidate (23b) reads $V_2 = \frac{1}{2}\varepsilon_2^2 + V_1$ with time derivative considering (25) and (22)

$$\dot{V}_2 = -c_1\varepsilon_1^2 + \varepsilon_1\varepsilon_2\hat{\theta}_u - d_1\varepsilon_1^2 + \varepsilon_1\tilde{z}_2 + \varepsilon_2(-b_1v_1 + \sigma(y)u - \hat{\delta}\ddot{y}_r - \frac{\partial\alpha}{\partial\boldsymbol{\mu}}\dot{\boldsymbol{\mu}} - \frac{\partial\alpha}{\partial y}(\hat{z}_2 + \omega_0 + \boldsymbol{\omega}^T\hat{\boldsymbol{\theta}}) - \frac{\partial\alpha}{\partial y}\tilde{z}_2) - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} - \tilde{\boldsymbol{\theta}}(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_2) \quad (26)$$

where $\boldsymbol{\tau}_2 = \boldsymbol{\tau}_1 - \frac{\partial\alpha_1}{\partial y}\boldsymbol{\omega}\varepsilon_2$.

The stabilizing function for the error dynamics (25) is

$$\alpha_2 = -\hat{\theta}_u\varepsilon_1 - c_2\varepsilon_2 - d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\varepsilon_2 + b_1v_1 + \frac{\partial\alpha_1}{\partial y}(\hat{z}_2 + \omega_0 + \boldsymbol{\omega}^T\hat{\boldsymbol{\theta}}) + \frac{\partial\alpha_1}{\partial\hat{\mathbf{z}}^T}\dot{\hat{\mathbf{z}}} + \frac{\partial\alpha_1}{\partial\boldsymbol{\xi}^T}\dot{\boldsymbol{\xi}} + \frac{\partial\alpha_1}{\partial\boldsymbol{\Psi}}\dot{\boldsymbol{\Psi}} + \frac{\partial\alpha_1}{\partial v_1}\dot{v}_1 + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}^T}\boldsymbol{\Gamma}\boldsymbol{\tau}_2 + \left(\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\delta}}\right)\dot{\hat{\delta}}.$$

Then, the nonlinear adaptive control law according to the algorithm is

$$u = \frac{1}{\sigma(y)}\left(-\hat{\theta}_u\varepsilon_1 - c_2\varepsilon_2 - d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\varepsilon_2 + b_1v_1 + \frac{\partial\alpha_1}{\partial y}(\hat{z}_2 + \omega_0 + \boldsymbol{\omega}^T\hat{\boldsymbol{\theta}}) + \frac{\partial\alpha_1}{\partial\hat{\mathbf{z}}^T}\dot{\hat{\mathbf{z}}} + \frac{\partial\alpha_1}{\partial\boldsymbol{\xi}^T}\dot{\boldsymbol{\xi}} + \frac{\partial\alpha_1}{\partial\boldsymbol{\Psi}}\dot{\boldsymbol{\Psi}} + \frac{\partial\alpha_1}{\partial v_1}\dot{v}_1 + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}^T}\boldsymbol{\Gamma}\boldsymbol{\tau}_2 + \left(\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\delta}}\right)\dot{\hat{\delta}} - \hat{\delta}\ddot{y}_r\right) \quad (27)$$

and the unified parameter estimator dynamics, following (24), is

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\tau}_2. \quad (28)$$

Replacing the adaptive control law (27) and the parameter estimator dynamics (28) into derivative (26), gives

$$\dot{V}_2 = -c_1\varepsilon_1^2 - c_2\varepsilon_2^2 - d_1\left(\varepsilon_1 - \frac{\tilde{z}_2}{2d_1}\right)^2 - d_2\left(\frac{\partial\alpha}{\partial y}\varepsilon_2 + \frac{\tilde{z}_2}{2d_2}\right)^2 + \left(\frac{1}{4d_1} + \frac{1}{4d_2}\right)\tilde{z}_2^2 - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}},$$

where the completion of squares is made by the damping terms $-d\varepsilon_1$, $-d_k(\partial\alpha_{k-1} / \partial y)^2\varepsilon_k$, $k = 2$. That is why, the sign indefinite terms including $\tilde{z}_2$ become sign definite. The stability of the whole system depends on the matrix $\mathbf{Q}$. If $\mathbf{Q} = diag(q_1, q_2)$ for $q_1 = w_1$, $q_2 = 1/4d_1 + 1/4d_2 + w_2$, and $w_1, w_2 > 0$, then

$$\dot{V}_2 = -c_1\varepsilon_1^2 - c_2\varepsilon_2^2 - d_1\left(\varepsilon_1 - \frac{\tilde{z}_2}{2d_1}\right)^2 - d_2\left(\frac{\partial\alpha}{\partial y}\varepsilon_2 + \frac{\tilde{z}_2}{2d_2}\right)^2 - \tilde{\mathbf{z}}^T\mathbf{W}\tilde{\mathbf{z}}$$

with $\mathbf{W} = diag(w_1, w_2)$, will be negative semi-definite with respect to $\tilde{\boldsymbol{\theta}}$. The asymptotic stability of the tracking and state observer errors follows from the La-Salle-Yoshizawa convergence theorem, which completes the nonlinear adaptive control design.

The approach is applied to a permanent magnet synchronous motor driven inverted pendulum system. With current mode PMSM control, the pendulum system original state space model has the form

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= -\theta_1\eta_2 - \theta_2\sin(y) + \theta_3u_1 + \theta_4u_1u_2 \end{aligned} \quad (29a)$$

$$y = \eta_1, \quad (29b)$$

where η_1 is the mechanical angle, η_2 – the rotor speed, u_1 – the control current i_d and u_2 – the control current i_q. The model parameters are $\theta_1 = \frac{b_f}{J}$, $\theta_2 = \frac{mgl}{J}$, $\theta_3 = \sqrt{\frac{3}{2}}\frac{n_p\psi_{pm}}{J}$, $\theta_4 = \sqrt{\frac{3}{2}}\frac{n_p\psi_{pm}(L_d - L_q)}{J}$ where m and l are the pendulum mass and length, J – the total moment of inertia, g – gravity acceleration, n_p – the number of pole pairs, ψ_{pm} – the magnets flux linkage magnitude, L_d, L_q are the motor inductances and b_f is the viscous friction coefficient. The control current $u_2 = i_q$ is asymptotically stabilized at zero, because the PMSM operates at lower than the rated speed, and will be further omitted in the paper.

The system (29) is transformed in an output feedback form (2)

$$\begin{aligned} \dot{x}_1 &= x_2 - \theta_1 y \\ \dot{x}_2 &= -\theta_2\sin(y) + \theta_3u_1 \end{aligned} \quad (30a)$$

$$y = x_1, \quad (30b)$$

with $\boldsymbol{\Phi}(y) = \begin{bmatrix} -y & 0 \\ 0 & -\sin(y) \end{bmatrix}$, $\boldsymbol{\varphi}(u) = \begin{bmatrix} 0 \\ u_1 \end{bmatrix}$ through the transformation $\boldsymbol{\eta} = \begin{bmatrix} x_1 \\ x_2 - \theta_1x_1 \end{bmatrix}$.

Since $f(y) = 0$, then $\boldsymbol{\xi} = \mathbf{0}$ and $\omega_0 = 0$. The filtered transformation dynamics defined in (6) is

$$\begin{aligned} \dot{\psi}_1 &= -b_1(\psi_1 - y) \\ \dot{\psi}_2 &= -b_1\psi_2 - \sin(y) \\ \dot{v}_1 &= -b_1v_1 + u_1 \end{aligned} \quad (31)$$

with the vector (5b) $\boldsymbol{\omega}^T = [(\psi_1 - y), \psi_2, v_1]$.

Hence, the AOCF (4) for the original system (29) is

$$\dot{z}_1 = z_2 + (\psi_1 - y)\theta_1 + \psi_2\theta_2 + v_1\theta_3 \quad (32a)$$

$$\begin{aligned} \dot{z}_2 &= b_1(\psi_1 - y)\theta_1 + b_1\psi_2\theta_2 + b_1v_1\theta_3 \\ y &= z_1 \end{aligned} \quad (32b)$$

with an adaptive nonlinear state observer (7)

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + (\psi_1 - y)\hat{\theta}_1 + \psi_2\hat{\theta}_2 + v_1\hat{\theta}_3 + n_1(y - \hat{y}) \\ \dot{\hat{z}}_2 &= b_1(\psi_1 - y)\hat{\theta}_1 + b_1\psi_2\hat{\theta}_2 + b_1v_1\hat{\theta}_3 + n_2(y - \hat{y}) \end{aligned} \quad (33a)$$

$$\hat{y} = \hat{z}_1, \quad (33b)$$

An observer error (8)

$$\begin{aligned} \dot{\tilde{z}}_1 &= \tilde{z}_2 + (\psi_1 - y)\tilde{\theta}_1 + \psi_2\tilde{\theta}_2 + v_1\tilde{\theta}_3 - n_1(y - \hat{y}) \\ \dot{\tilde{z}}_2 &= b_1(\psi_1 - y)\tilde{\theta}_1 + b_1\psi_2\tilde{\theta}_2 + b_1v_1\tilde{\theta}_3 - n_2(y - \hat{y}) \end{aligned}$$

and adaptive parameter estimator dynamics (12) with an observer tuning function (13) $\boldsymbol{\tau}_0 = [\psi_1 - y, \psi_2, v]^T(y - \hat{y})$.

Following the procedure, the next tuning functions are computed

$$\boldsymbol{\tau}_1 = \begin{bmatrix} (\varepsilon_1 + \tilde{z}_1)(\psi_1 - y) \\ (\varepsilon_1 + \tilde{z}_1)\psi_2 \\ \varepsilon_1\varepsilon_2 + \tilde{z}_1v_1 \end{bmatrix}, \quad \boldsymbol{\tau}_2 = \begin{bmatrix} (\varepsilon_1 + \tilde{z}_1 + \varepsilon_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_1))(\psi_1 - y) \\ (\varepsilon_1 + \tilde{z}_1 + \varepsilon_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_1))\psi_2 \\ \varepsilon_1\varepsilon_2 + (\tilde{z}_1 + \varepsilon_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_1))v_1 \end{bmatrix}.$$

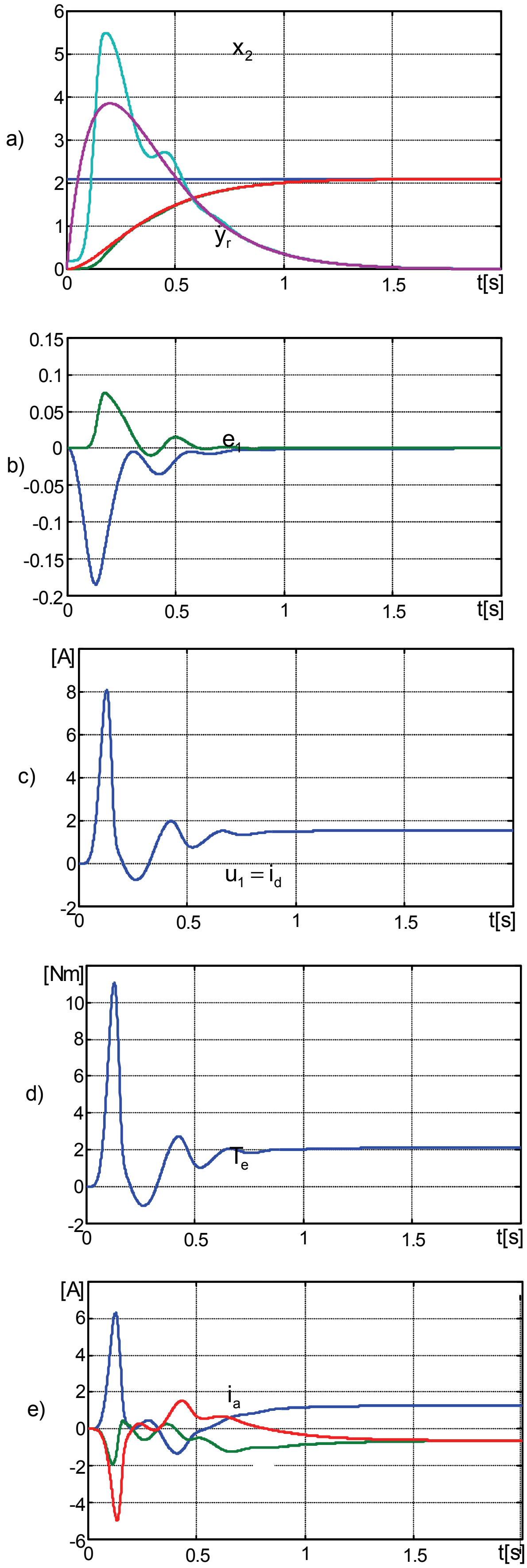

Figure 1. System output, error, control, torque and abc currents responses

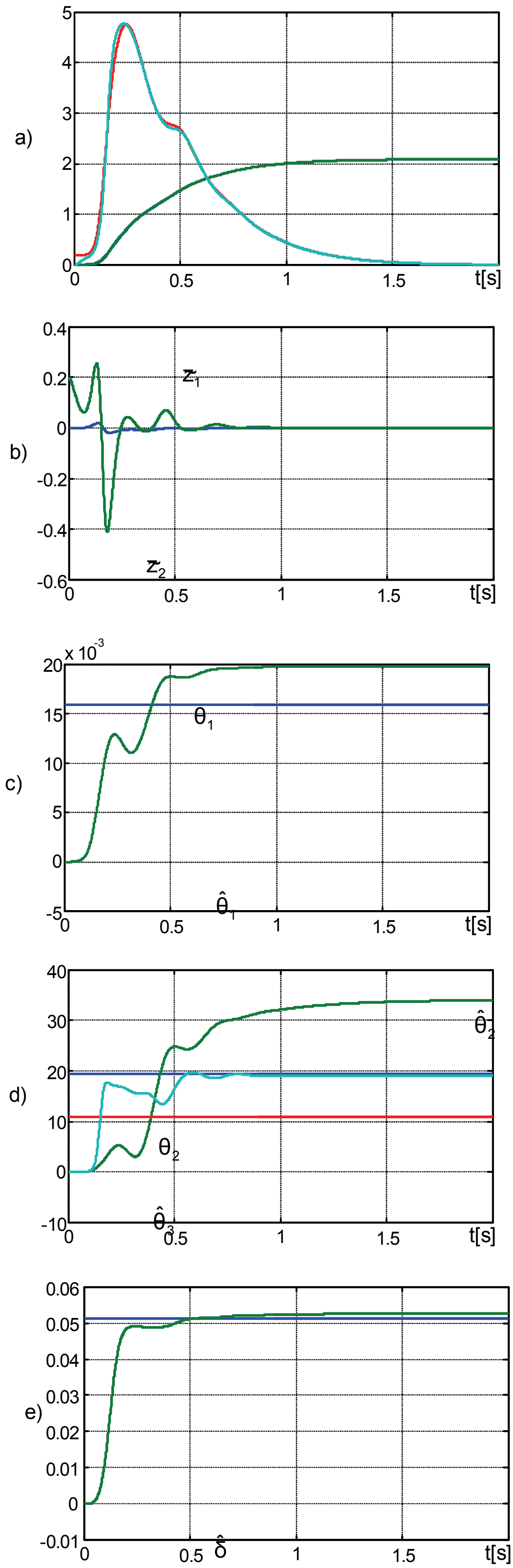

Figure 2. z observer and parameter estimators responses

The tuning functions $\boldsymbol{\tau}_\rho$, $\rho = 0,1,2$ define the final parameter estimator dynamics (28) and (21)

$$\dot{\hat{\theta}}_1 = \gamma_1(\varepsilon_1 + \tilde{z}_1 + \varepsilon_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_1))(\psi_1 - y) \tag{34a}$$

$$\dot{\hat{\theta}}_2 = \gamma_2(\varepsilon_1 + \tilde{z}_1 + \varepsilon_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_1))\psi_2 \tag{34b}$$

$$\dot{\hat{\theta}}_3 = \gamma_3(\varepsilon_1\varepsilon_2 + (\tilde{z}_1 + \varepsilon_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_1))v_1) \tag{34c}$$

$$\dot{\hat{\delta}} = -sign(\theta_3)\gamma_\delta(\dot{y}_r - \hat{z}_2 - (c_1 + d_1)\varepsilon_1 - (\psi_1 - y)\hat{\theta}_1 - \psi_2\hat{\theta}_2))\varepsilon_1 \tag{34d}$$

The final complete adaptive control law is

$$\begin{aligned} u_1 = & -c_2\varepsilon_2 + b_1v_1 + (c_1 + d_1)\dot{y}_r\hat{\delta} + \hat{\delta}\ddot{y}_r - d_2\varepsilon_2\hat{\delta}^2(c_1 + d_1 - \hat{\theta}_1)^2 - \varepsilon_1\hat{\theta}_3 + \hat{\delta}\dot{\hat{\theta}}_1(y - \psi_1) + b_1\hat{\delta}\hat{\theta}_1(\psi_1 - y) - \\ & \hat{\delta}\dot{\hat{\theta}}_2\psi_2 - \dot{\hat{\delta}}((c_1 + d_1)\varepsilon_1 - \dot{y}_r + \hat{z}_2 + \hat{\theta}_1(\psi_1 - y) + \hat{\theta}_2\psi_2) - \hat{\delta}(c_1 + d_1 - \hat{\theta}_1)(\hat{z}_2 + \hat{\theta}_1(\psi_1 - y) + \\ & \hat{\theta}_2\psi_2 + \hat{\theta}_3v_1) - \hat{\delta}(n_2\tilde{z}_1 + b_1(\hat{\theta}_1(\psi_1 - y) + \hat{\theta}_2\psi_2 + \hat{\theta}_3v_1)) + \hat{\delta}\hat{\theta}_2(b_1\psi_2 + \sin(y)). \end{aligned} \tag{35}$$

providing asymptotic stability of the tracking error.

5. Simulation and System Time Responses

The equations of system (29), (31), (32), (33), and (34) with nonlinear adaptive control (35) are simulated. The PMSM synchronous servomotor used is Lenze MDSKS071-03 with the following parameters: nominal power $P_N = 2$ kW, nominal torque $T_N = 5.7$ Nm, rated speed $\omega_N = 356$ *rad/s* , nominal AC voltage $U_N = 330$ V, nominal current $I_N = 4.2$ A, torque constant $k_T = 1.37$ Nm/A, stator resistance $R_s = 3.4$ Ω, moment of inertia $J = 6 \times 10^{-4}$ $kg.m^2$, stator inductance $L_s = 10.6$ *mH*, number of pole pairs $n_p = 3$. The pendulum attached to the shaft is determined by the mass $n = 0.5$ kg, link length $I = 0.5$ m, and the gravity constant $g = 9.81 ms^{-2}$. The closed-loop adaptive system including the nonlinear adaptive state and the parameter observers with adaptive tuning functions control is dynamically simulated at initial conditions $\mathbf{x}_0 = [0, 0.2]^T$, $\mathbf{z}_0 = [0, 0.2]^T$, $\hat{\mathbf{z}}_0 = [0, 0]^T$, $\hat{\boldsymbol{\theta}}_0 = [0, 0, 0]^T$, $\hat{\delta}_0 = 0$. The reference trajectory is the state vector of a second order linear system with a double real pole $p_{12} = -5$. The adaptive tuning functions control law is defined by the parameters $c_1 = 10$, $c_2 = 1$, $d_1 = d_2 = 10$. The observer gain matrix is $N = [80, 1600]^T$ which sets a double pole –40 for the observer dynamics. The parameter estimator gains are $\gamma_1 = 10$, $\gamma_2 = \gamma_3 = 10^5$, $\gamma_\delta = 0.5$. *Figure 1a* displays the tracking of the reference trajectory $y_r(t)$, $\dot{y}_r(t)$ by the nonlinear closed loop adaptive system for a reference input $r = 2\pi/3$ with tracking errors shown in *figure 1b*. The motor torque current in rotating dq coordinates $u_1 = i_d$, the electromagnetic torque T_e and the respective currents in abc coordinates i_a, i_b, i_c, are depicted in *figures 1c–e*. The i_q current is asymptotically stabilized at zero. The adaptive *state* observer evolution with its state estimation errors $\tilde{z}_1$ *and* $\tilde{z}_2$ are displayed in *figures 2a–b*. The dynamics of the estimated parameters are given in *figures 2c-e*. The tracking of the reference trajectory is achieved by the closed loop adaptive system with the prescribed overdamped performance specification. Thus, the system time responses have confirmed the tracking errors convergence to zero with Lyapunov stable parameter estimates, and the asymptotic stability of the nonlinear adaptive state observer.

6. Conclusion

The paper has presented a nonlinear adaptive observer and parameter estimator design for nonlinear adaptive tracking control of a synchronous motor driven inverted pendulum. The adaptive tracking control is designed by the adaptive tuning functions approach. The nonlinear adaptive observer is designed in an adaptive observer canonical form via the filtered transformation. The adaptive parameter estimator is designed by the direct Lyapunov method. Its dynamics is chosen to provide asymptotically stable adaptive observer error with the cost of only Lyapunov stable parameter estimates. The closed loop nonlinear adaptive tracking control system, incorporating an asymptotically stable nonlinear adaptive state observer and Lyapunov stable parameter estimator achieves trajectory tracking with the prescribed overdamped performance specifications.

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