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Optimal Control of Mobile Agents for Monitoring of Points on a Network

V. Sgurev, S. Drangajov

Key Words: *Mobile agents' path control; network flow optimization; cycles on graphs.*

Abstract. *This paper concerns the problems of finding optimal trajectories between nodes on a network, which must be periodically surveyed, and probably serviced. It is shown, that such trajectories may be generated if optimal Hamiltonian cycles are used between the separate network nodes under inspection. It is known that the Hamiltonian path problem is NP-complete, but an edge decomposition of the network is proposed. This is performed by reducing in a particular way to network flow circulations. The requirements and the equations for describing such circulation are pointed out. Defining of the optimal circulations of the mobile agents is reduced to network flow programming problems. A numerical example is presented for solving a similar class of monitoring problems by mobile agents.*

1. Preliminaries

The problem of monitoring appointed nodes of a network has been topical since ancient times for security of military camps, large ware houses areas, civil objects, etc. [2,3,7]. With the accelerating development of mathematics in the last centuries, and especially of the graph theory, planning of the route of inspection of the objects is a matter of strict formal rules. Herein a mobile agent is considered a unit – a robot carrying some measurement or watch devices (visual or thermo vision cameras, gas analyzers, etc.), human military patrol, police patrol within a town region, inspection of corporative ware houses fields, preventive fire protection, environmental control, etc. [1,8,10]. The common feature between all these agents is that they move along a pre assigned circular route over a network and inspect specified points and in case of trouble undertake actions and inform the higher control level. In case of emergency new agents may be added and the schedule - changed.

In large industrial sites, like power plants, chemical plants, etc., the monitoring is automatically carried out by mobile robots equipped with the respective measurement appliances. They circulate on the network along given route sectors. The difference in specifying the optimal route, compared to, say a military or eco patrol is only in the real scale of time and distance. In this paper we try to use only relative units, so that the model may be applicable for any mobile inspecting agent.

When executing this task, some problems arise in the minimization of the mobile agents' number, necessary to observe the tour requirements, keeping at the same time the condition that each point under control is being monitored in a time gap not greater than the prescribed one. This, on the other hand, results in increasing the necessary number of mobile agents to observe the respective time requirements, i.e., a problem arises to define the optimal control of the mobile agents, observing contradictory constraints. The problem may be solved through an appropriate network flow model for finding the optimal flow circulation.

2. Formal Presentation of the Problem and the Graph Model

2.1. The Problem

A connected network with a finite number of nodes and arcs between them is given. The predefined nodes must be periodically visited by a mobile agent (human or robotic), inspected and probably serviced. The maximum time between the inspections of each node is defined. The value (time, cost or whatever else) for passing along the arc of each agent is defined.

Find: a) the minimum number of mobile agents which could accomplish the task, thus defined; b) the optimal routes of the mobile agents, so that the value of their tour is minimal.

2.2. The Graph Model

For solving the problem thus defined, it is expedient the network of the possible paths and branches in the movement of the mobile agents to be represented as an oriented graph $G(X,U)$ [4] with a set of nodes $X=\{x_1, x_2, ..,x_n\}$ and a set of arcs $U=\{u_1, u_2, ..,u_m\}$.

The following denotations will be used:

$u_k = (x_i, x_j) \in U$ – a single arc;

$I(U) = \{(i,j)\} / (x_i, x_j) \in U\}$ – the set of all indices of pairs of existing arcs;

$X_0 \subset X$ – the set of all points on the network subject to control and inspection;

$X' = X \setminus X_0$ – the set of all intermediate points for branching on the route of the mobile agents;

Γ_x^1 – direct mapping of node $x \in X$;

Γ_x^{-1} – reverse mapping of node $x \in X$;

$g_{ij} = g(x_i, x_j)$ – time (or value) for passing of the mobile agent along the oriented arc (x_i, x_j) from node x_i to x_j;

$Q(x)$; $x \in X_o$ – maximum allowed time between two consequent inspections by the mobile agent of the object $x \in X_o$;

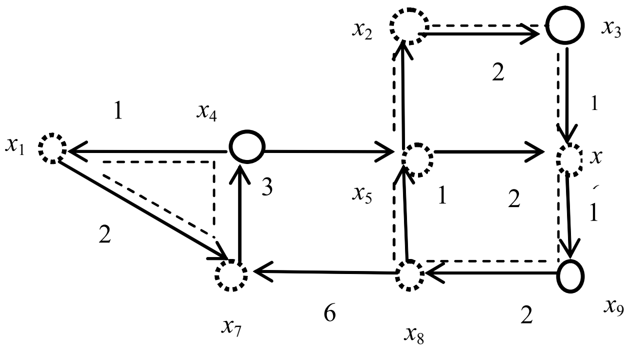

Figure 1

$f_{ij} = f(x_i, x_j); (i, j) \in I(U)$ – a non-negative arc flow function.

The time, necessary for mobile agent stay for observation of the objects being controlled $x \in X_o$, is included in the value g_{ij} for the respective arc (x_i, x_j) and will not be considered separately from it.

Under the prerequisite thus made, the movement of each mobile agent for execution of the assigned tasks may be most generally considered as a network flow circulation [5], at which this movement may be in particular interpreted through a cyclic network flow of value one. The flow conservation equations are observed for each node $x \in X$, according to which the incoming flow in the node is equal to the outgoing, i.e.,

$$\sum_{j \in \Gamma_i^1} f_{ij} - \sum_{j \in \Gamma_i^{-1}} f_{ji} = 0; x_i \in X;$$

A specific feature of the problem for the mobile agents being considered is that during the circulation of one or a group of agents, all nodes, corresponding to the objects under surveillance $x \in X_o$, must necessarily be in the cycle of at least one agent, i.e., they must be periodically visited. This may performed by the following requirement:

for each $x \in X_o$

$$\sum_{j \in \Gamma_i^1} f_{ij} = 1 .$$

It is of course necessary the functional $\sum_{(i,j) \in I(U)} g_{ij} f_{ij} \to \min$ to be minimized, which guarantees minimal run of the mobile agents, at that with minimal number of the latter.

So the problem for optimal control of circulating mobile agents, defined in the present paper may be reduced to the following integer (binary) programming problem

(1) $L = \sum_{(i,j) \in I(U)} g_{ij} f_{ij} \to \min;$

under constraints: for each $x \in X_o$

(2) $\sum_{j \in \Gamma_i^1} f_{ij} - \sum_{j \in \Gamma_i^{-1}} f_{ji} = 0;$

(3) $\sum_{j \in \Gamma_i^1} f_{ij} = 1$ for $x \in X_o$;

(4) $f_{ij} = C_{ij}$, for each $(i,j) \in I(U)$,

where $f_{ij} = 0$ or 1 for each $(i,j) \in I(U)$, and C_{ij} is the capacity of the arc $(x_i, x_j) \in I(U)$.

It is not compulsory a unique Hamiltonian cycle to be obtained through problem (1) to (4) for the route of one agent – this would be a welcome case. We remind once again that the Hamiltonian path problem is NP-complete. It is also not compulsory the cycles to cover all nodes of X. But **it is compulsory** that all inspected nodes X_o enter the cycles, i.e., as mentioned in the abstract we use edge decomposition.

If the number of the available mobile agents is less than the optimal number obtained by the problem from (1) up to (4), then through additional constraints on the arc flow functions, the optimal number of agents may be reduced to their target number, observing to a great extent the optimal parameters. Such a decrease should also be done, if any inspection encirclement cycle for the points being serviced, is of greater time or value than the above mentioned.

3. Numerical Examples

Two numerical examples are presented, which were solved by the WebOptim software package [6].

3.1. Example 1

An oriented graph with 9 nodes $|X| = 9$) and 12 arcs – $|U| = 12$ is shown in *figure 1*. The objects under inspection are three

(5) $X_0 = \{x_3, x_4, x_9\}$.

The corresponding nodes are circles with thick lines. The other nodes are dotted circles.

Table 1 shows the arc ratings $\{g_{ij}\}$ of the total loss of time (or value) for the inspection visit of the nodes from X_0

In order to define the optimal cycles, i.e., the optimal number of mobile agents for observation of the check points from X_0, it is necessary to solve the integer programming problem given by (1) up to (4), taking into concern the network from *figure 1*, the check points from (5) and the data from *table 1*. The objective function L and the equa-

Table 1

Index of the rating	$g_{1,7}$	$g_{2,3}$	$g_{3,6}$	$g_{4,1}$	$g_{4,5}$	$g_{5,2}$	$g_{5,6}$	$g_{6,9}$	$g_{7,4}$	$g_{8,5}$	$g_{8,7}$	$g_{9,8}$
Arc rating	2	2	1	1	5	4	2	1	3	1	6	2

Table 2

Index of the rating	$f_{1,7}$	$f_{2,3}$	$f_{3,6}$	$f_{4,1}$	$f_{4,5}$	$f_{5,2}$	$f_{5,6}$	$f_{6,9}$	$f_{7,4}$	$f_{8,5}$	$f_{8,7}$	$f_{9,8}$
Arc rating	1	1	1	1	0	1	0	1	1	1	0	1

Table 3

Arc flow	$f_{1,7}$	$f_{2,3}$	$f_{3,6}$	$f_{4,1}$	$f_{4,5}$	$f_{5,2}$	$f_{5,6}$	$f_{6,9}$	$f_{7,4}$	$f_{8,5}$	$f_{8,7}$	$f_{9,8}$
Arc flow value	0	1	1	0	1	1	0	1	1	0	1	1

tions (1) up to (7) may be put down in the following form:

$$L = 2f_{1,7}+2f_{2,3}+f_{3,6}+f_{4,1}+ 5f_{4,5}+4f_{5,2}+ 2f_{5,6}+f_{6,9}+ 3f_{7,4}+f_{8,5}+ + 6f_{8,7}+ 2f_{9,8}\rightarrow \min;$$

$f_{1,7} - f_{4,1} = 0;$

$f_{2,3} - f_{5,2} = 0;$

$f_{3,6} - f_{2,3} = 0;$

$f_{4,1} + f_{4,5} - f_{7,4} = 0;$

$f_{5,2} + f_{5,6} - f_{4,5} - f_{8,5} = 0;$

$f_{6,9} - f_{3,6} - f_{5,6} = 0;$

$f_{7,4} - f_{1,7} - f_{8,7} = 0;$

$f_{8,5} + f_{8,7} - f_{9,8} = 0;$

$f_{9,8} - f_{6,9} = 0;$

$f_{3,6} = 1;$

$f_{4,5} - f_{4,1} = 1;$

$f_{9,8} = 1;$

$f_{i,j} = 0$ or 1; where $(i , j) \in I(U)$.

The program package WebOptim [6] was used to solve the problem. The solution was found in 6 iterations on a middle class laptop. The optimal solution is given in *table 2*.

The arcs corresponding to the optimal solutions from *table 2* are shown by dotted lines in *figure 1*. It follows from this that two mobile agents are necessary to observe the three objects under inspection $\{x_3, x_4, x_9\}$ which travel along two optimal cycles C_1 and C_2. At that the first mobile agent inspects object x_4 and the second cycle C_2 – the two objects x_3 and x_9 through periodical circulation.

(6) $C_1 = \{x_4\} = \{(x_4, x_1), (x_1, x_7), (x_7, x_4)\}$;

(7) $C_2 = \{x_3, x_9\} = \{(x_3, x_6), (x_6, x_9), (x_9, x_8), (x_8, x_5), (x_5, x_2), (x_2, x_3)\}$.

It follows from the comparison of the objective function L from both *tables 1* and *2* that six units are necessary for the first cycle $C_1 = \{x_4\}$, and for the second one $C_2 = \{x_3, x_9\}$ – 11 units, or 17 units total. It is evident that under the source requirements set no more economic service of the three objects being inspected is possible.

In case that too frequent inspection of the three objects is not necessary, a consequent check may be carried out for all of them by a single mobile agent. The next example illustrates this opportunity.

3.2. Example 2

Let the same graph from *figure 1* be defined with arc flow rating from *table 2,* the three check points from (5) and the respective equations from (1) up to (4). To reach a single optimal cycle, an additional requirement is added

(8) $f_{4,1} = 0$.

The optimal solution of this new problem is shown in *table 3*.

In *figure 2* the arcs corresponding to the new optimal

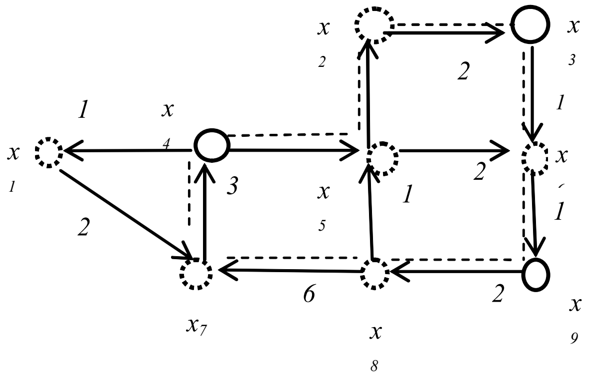

Figure 2

solution are shown by dotted lines. They define a single Hamiltonian cycle in which the two arcs – {(x_4, x_1) and (x_1, x_7) are not included.

This means that for service of the three objects under inspection from X_0, a single mobile agent will be necessary which will inspect them from x_3 to x_9, from x_9 to x_4 and from there – back to x_1, according to the cycle from *figure 2*.

(9) $C_1 = \{x_3, x_9, x_4\} = \{(x_3, x_6), (x_6, x_9), (x_9, x_8), (x_8, x_7), (x_7, x_4), (x_4, x_5), (x_5, x_2), (x_2, x_3)\}.$

It follows from the objective function L and *tables 1* and *3* that for an inspection cycle of the check points along the cycle from (9), 24 units will be necessary. More economic inspection around the objects being observed is not possible by a single mobile agent.

The comparison between the two examples 1 and 2 shows that the total expenses per a round in the second case (a single mobile agent) are greater than when using two mobile agents. But at the expense of this initial investment expenses, and therefore, the operating expenses of two agents are greater in the first example – with two mobile agents. In this case a decision must be made on the next higher management level in the design process of the entire monitoring system.

4. Summing up

A method is proposed for optimal control of mobile agents for monitoring and inspection of nodes on a connected network of arbitrary configuration. The nature of the agent is of no importance – it may be a mobile robot, equipped with the appropriate appliances in a large production sites, or large ware houses, a military patrol watching points in a big camp [10], a police patrol circulating in a district, etc. The common feature is that the agent must move along prescribed paths and must observe the requirements for periodical frequency of inspections.

The optimal trajectories of these mobile agents are defined by appropriate flow circulations on the network, at which two contradictory requirements must be kept: using the minimal possible number of mobile agents, guaranteeing at that the maximum admissible time between two consecutive inspections of the respective node by the mobile agent. The method is reduced to solving a specific class of problems of network flow binary optimization. These problems are NP-complete.

Two numerical examples are given which illustrate the abilities of the methods and algorithms being presented.

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*Academician **Vassil Sgurev** was born in Parvomay in 1936. He had received his high education in Sanct Peterburg, Russia. His career had been dedicated on the science. In the Bulgarian Academy of Sciences academician Vassil Sgurev had walked the hard professional way from Associate Researcher to Associate Professor, Ph.D. and D.Sc., Corresponding Member to the highest position – Academician. The research achievements of him are published in 340 science articles, 8 monographs, 30 patents and inventions had been created, including the first computer systems of industrial processors ASTRA and TRASI controlling. He has been a supervisor of 15 defended Ph.D. students. Academician Vassil Sgurev had read lectures in many universities and scientific organizations in Bulgaria and abroad. He had been a chairman on the program committees of 20 international and 25 national scientific conferences and symposiums. Academician Vassil Sgurev develops active organization and administrative activity: in the Bulgarian Academy of Sciences – a Head of Departments, Director of Institute of Engineering Cybernetics and Robotics (1983 – 1990) and Institute of Informatics (1990 – 1993); Scientific Secretary of the Bulgarian Academy of Sciences; Member of the Scientific Councils High Testimonial Committee of Bulgaria; Member of Editorial Boards of many scientific journals; Member of prestige scientific organizations. He has been a President of the Federation of the Scientific Engineering Unions in Bulgaria since 2000. The scientific and research activities of Academician Vassil Sgurev confirms him as an eminent scientist, organizer and leader in the area of the Technical Cybernetics, Automation, Operation Research, Artificial Intelligence, Informatics and Information Technologies.*

Contacts:
Department Inteligent Systems
Institute of Information and Communication Technologies
Bulgarian Academy of Sciences
Acad G. Bonchev St., bl. 25A, 1113 Sofia
e-mai: sgurev@bas.bg

***Stanislav Drangajov** was born in 1953 in Haskovo, Bulgaria. M.S. degree in 1979 in mechanical engineering from Technical University – Sofia. Researcher in Institute of Engineering Cybernetics and Robotics – BAS since 1979 and its later heirs till the present Institute of Information and Communication Technologies. Interests – operations research, especially network flows and Markov decision processes. Rich expertise in industrial implementations of optimization methods.*

Contacts:
Bulgarian Academy of Sciences
Acad. G. Bonchev St., bl. 2, 1113 Sofia
e-mail: sdrangajov@gmail.com
http://scholar.google.com/